Spatially Irregular Space-Waveform Coded CPFSK with Full Rank Optimized Waveforms

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Abstract—A nonlinear modulation, the constant envelope CPM class being of particular interest, has multi-dimensional signal waveforms. We introduce a spatial code for multi-antenna CPFSK system using a different waveform (constellation) space on each antenna. The 2-dimensional space-waveform codeword spans an antenna (spatial) and constellation (waveform) domains. We optimize the waveforms for rank and determinant criterion. The resulting space-waveform code exhibits a full rank without any need of temporal domain encoding. Numerical results are presented for a selected set of CPFSK class of signals and 2-antenna transmitter. In this particular case, the procedure leads to an elegant implementation by Spatially Mirrored Tilted Phase CPFSK. The presented scheme is well suited for the concatenation with outer scalar temporal domain code.

I. INTRODUCTION

A. Background And Related Work

The theory of linear space-time coded modulation for Multiple-Input Multiple-Output (MIMO) channels and a corresponding signal processing is widely covered in the literature, e.g. [1], [2], [3], [4]. Space-Time Codes (STCs) applied to nonlinear modulations started to attract the attention of researchers only recently and the corresponding theory covers the area only sparsely. Some performance aspects of nonlinear STCs, like information capacity, code design for selected channels, and decoder performance were investigated in e.g. [5], [6], [7], [8], [9], [10], [11], [12].

Nonlinear CPM (Continuous Phase Modulation) modulations have a range of very useful properties and provide a very attractive alternative to linear modulations. They are inherently resistant to nonlinear distortion (caused by e.g. C-class amplifier) at the transmitter. As a consequence of the modulation nonlinearity, the dimensionality of the waveform per transmit antenna is higher than one. This opens more dimensions of the channel and also allows to decouple a particular waveform (pulse) shape from its second order properties.

B. Goals of this Paper and Contributions

The paper concentrates on the multidimensional nature of the CPM waveforms (signal space constellation) and develops a technique to use this in the coded signal construction for a MIMO channel. In simple terms, we try to develop a waveform construction (at the level of CPM’s memoryless waveform mapper) that would add full diversity regardless of the outer spatial-temporal processing—outer space-time code and/or CPM’s continuous phase encoder. This is somewhat similar to the philosophy of Alamouti (or any orthogonal design) construction concatenated to the outer code.

We start with the principle of the coding in spatial, temporal and waveform domains. The waveform domain coding (we can view this also as a multi-D constellation shaping) is shown to be an approach of designing the waveforms with certain constraints (e.g. constant envelope, continuous signal) and also having desired performance parameters (e.g. the diversity and the coding gain). It could be utilized independently of the spatial and/or temporal domain. A space-time-frequency coding is shown to be a very special case with no constraints in the waveform domain.

We take a simple binary full response irregular 2-channel CPFSK and optimize its waveforms for the determinant and full rank criterion. The result of the optimization leads to a spatially irregular modulation that can be elegantly implemented as Spatially Mirrored Tilted Phase CPFSK. The resulting scheme is shown to have a full diversity rank. It is achieved by the waveform dimensionality expansion within an approximately constant bandwidth w.r.t. the reference MSK scalar modulation.

II. SYSTEM MODEL AND DEFINITIONS

A. Multi-Channel Irregular CPM Modulation in MIMO Channel

We consider a multiple transmit $N_T$ and receive $N_R$ antenna system. Each transmit antenna transmits a CPM modulated signal. The CPM signals do not need to have uniform properties—IMC CPM (Irregular Multi-Channel CPM). We allow the signals on individual antennas to have a general modulation index $\kappa$, an arbitrary continuous phase function $\beta(\cdot)$ (generally a nonlinear function of data), and a symbol alphabet size $M_c$. The complex envelope signal on $i$th antenna is

$$s_i(t) = \exp\left(j2\pi\kappa_n\sum_n \beta(c_{ni}, t - nT_S)\right)$$

(1)

where $T_S$ is the symbol period and $c_{ni} = c_{ni}(d_n)$ the coded symbol depending on the data symbol $d_n$.

In this paper, we restrict ourselves to binary alphabet ($M_c = 2$, $c_{ni} \in \{1, 2\}$) and CPM with linear full-response phase
function, i.e. CPFSK. We define a proto-phase function

\[ \beta_0(t) = \begin{cases} 0; & t < 0 \\ t/(2T_S); & t \in [0, T_S) \\ 1/2; & t \geq T_S \end{cases} \]

The irregular phase functions are

\[ \beta_i(c_{ni}, t) = \begin{cases} \alpha_{i1}\beta_0(t); & \text{for } c_{ni} = 1 \\ \alpha_{i2}\beta_0(t); & \text{for } c_{ni} = 2 \end{cases} \]

where \( \alpha_{i1} \in [-1, 1] \) are constants controlling the instant frequency for individual symbols and antennas. Notice that the choice of \( \alpha_{i2} = 1 \), \( \alpha_{i2} = -1 \) corresponds to a standard binary regular CPFSK modulation.

A canonic Finite State Machine (FSM) representation of the IMC CPFSK modulator is

\[ s_i(c_{ni}, t) = e^{j2\pi\kappa_i\beta_i(c_{ni}, t)} \psi_{n_i}(t) \left( \Upsilon(t) - \Upsilon(t - T_S) \right) \]

where \( \Upsilon(t) \) is the unit step function and \( \psi_{n_i} \) is the phase state of the modulator. It obeys to the state update equation

\[ \psi_{n+1,i} = \left( \psi_{n,i} + j2\pi\kappa_i\alpha_{i1} \right) \mod 2\pi \]

\[ \psi_{n+1,i} = \left( \psi_{n,i} + j2\pi\kappa_i\alpha_{i2} \right) \mod 2\pi \]

where \( c_{ni} = 1 \), \( c_{ni} = 2 \).

The modulation satisfies the Nyquist criterion.

The phase state FSM constitutes so called Continuous Phase Encoder (CPE). The Memoryless Waveform Mapper (MWM) maps the CPE encoded data onto channel waveforms. For the full response CPM, the CPE influences only the phase rotation which cannot affect the distance properties. The prototype MWM functions (neglecting a temporal index and CPE phase rotations) are

\[ s_0(c_i, t) = e^{j2\pi\kappa_i\beta_i(c_i, t)} \left( \Upsilon(t) - \Upsilon(t - T_S) \right) \]

The modulated signal passes through Rayleigh flat fading block-constant MIMO channel with AWGN. The channel coefficients \( h_{ji} \) (between \( i \)th Tx and \( j \)th Rx antenna) are assumed to have a unity variance. The AWGN noise has \( \sigma_n^2 = 2N_0 \) variance per signal-space (orthonormal basis) dimension.

### III. Waveform Domain Coding

#### A. Spatial, Temporal and Waveform Domain

A channel encoding process is a procedure mapping the discrete data onto continuous signal waveforms that are transmitted over the MIMO channel. The transmitted signals can be generally represented using the 3-dimensional indexing of the signal space basis (the signal domains). Each index of the particular domain represents a signal space basis sequential index within that domain. (1) The spatial domain corresponds to the transmit antenna index. (2) The temporal domain corresponds to the sequential number of the channel symbol as they are transmitted in the time sequence. (3) The waveform domain describes the signal subspace corresponding to one channel symbol in the time sequence and on one transmit antenna.

The channel signal at \( i \)th Tx antenna, \( n \)th time domain slot, and \( k \)th waveform dimension is

\[ s_{i,n,k}(d_{n,t}) = q_{i,n,k}(d_n)g_{i,k}(t - nT_S) \]

where \( d_n \) is the data symbol, \( q_{i,n,k}(d_n) \) is the signal space codeword symbol and \( g_{i,k}(t) \) is the channel continuous waveform basis function. The spatial index is \( i \in \{1, \ldots, N_T\} \), the temporal index is \( n \in \mathbb{Z} \), and the waveform index is \( k \in \{1, \ldots, N_W\} \).

The waveform subspace has (complex) dimensionality equal to 1 for arbitrary linear modulation (e.g. PSK, QAM). Therefore, most of the coding technique for MIMO channel fits under Space-Time label. Unlike for the linear modulation, the waveform dimensionality of the nonlinear modulation is \( N_W > 1 \).

A very special, however extremely popular example of multi-dimensional channel symbol waveforms is the OFDM modulation. The OFDM is of course a nonlinear modulation when viewed from the perspective of the mapping between the composite (over all sub-carriers) data symbol and the resulting modulated signal. The OFDM waveform space has a very special form. It forms orthogonal frequency separated subspaces of individual sub-carriers.

The waveform domain can be used to form a 3-D space-time-waveform codeword. The waveform codeword can be also viewed as multi-D constellation point shaping. This codeword construction has been so far considered only for the very special case of OFDM formatted waveform subspaces and it is known as a Space-Time-Frequency coding.

#### B. Waveform Subspace Characteristics

The work in this paper is a first step to generalization of the 3-D space-time-waveform codeword design for arbitrary nonlinear modulation (CPM being of a special interest).

The special feature of the mutual orthogonality of OFDM waveform subspaces is not assumed anymore. This fact itself is not crucial. Of course, the orthogonality simplifies the receiver processing but it is not directly related to the performance optimality. The temporal domain orthogonality is usually referred as the Nyquist property and it is required in order for the receiver metric folding condition to hold.

The sub-carrier subspace orthogonality was an obvious design goal for the OFDM signals that helped to split the channel into narrow flat fading chunks. On the other side, the construction of the waveform space of CPM modulation is fully obeyed to the constant envelope property. This makes the signal resistant to the nonlinear distortion. Multiple waveform dimensions open additional degrees of freedom and allow essentially to decouple second order signal properties from the waveform shape.

Irregularity of the CPMs at individual transmit antennas allows to further expand the overall waveform dimensionality and thus to increase the dimensionality of the 3-D codeword space. The dimensionality is asymptotically closely related to the bandwidth of the signal \( B \ll N_c \). But fortunately it holds only asymptotically. The CPM signal bandwidth usually grows much more slowly for small dimensionalitys. In order to keep the codeword space at (at least approximately) a
constant total bandwidth, we must properly adjust the parameters of individual CPMs (typically the modulation index). Fig. 1 demonstrates several example situations of space-time-waveform codeword construction.

C. Waveform and Symbol Layer Coding

1) Two layer design: There are essentially two layers of the code design procedure when designing the coded signals for a continuous time and value domain channel.

2) Waveform shape constraint (i.e. we want to have a given particular shape of the signal waveform, e.g. constant envelope). We could have applied an orthogonal Alamouti design [13] over the waveform shape constraint (i.e. we want to have a given particular waveform shape). This matching can have various targets, either simply helping with a given performance feature or adding a new quality (e.g. the diversity).

This paper concentrates on the second layer of the coded signal design. In order to show the benefits, we present the following example design goal—the diversity waveform coding.

2) Diversity waveform coding: Assume a goal of designing a full rank rate 1 diversity code for \( N_T = 2 \) Tx antennas with a waveform shape constraint (i.e. we want to have a given particular shape of the signal waveform, e.g. constant envelope). We could have applied an orthogonal Alamouti design [13] over spatial and temporal domains or over spatial and waveform domains. In both cases, in order for the Alamouti code to work, we would have needed orthogonal \( g_{i,k} \) waveform (Nyquist) or waveform domain (e.g. OFDM). Here, the waveform domain is formed of orthogonal functions leaving no other degree of freedom for the design.

An assumption of general nonlinear modulation with multi-D general waveform space allows additional degree of freedom for the coded signal design. We can create a tailor-made, matched or optimized waveforms suited for the outer \( g_{i,n,k} \) code. This matching can have various targets, either simply helping with a given performance feature or adding a new quality (e.g. the diversity).

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D. Dimensionality Degrees of Freedom

The design of the full rank diversity code requires that the signal space of all codewords differences \( \Delta g_{i,n,k} \) spans the subspace of the dimensionality equal to the number of the Tx antennas, \( N_I = N_T \) (the rank criterion [1]). An Alamouti code creates the degrees of freedom needed to construct the independent signals by a very specific concatenation of the fragments (symbols) (in time/waveform domain) with no additional constraints applicable. Unfortunately, this is not allowed when we are constrained to keep given waveform shape (e.g. continuous signal with a constant envelope).

We introduce additional degrees of freedom allowing us to design the diversity waveform code by a dimensionality expanding of the waveform space. Of course, in order to keep fair conditions, we must expand the dimensionality while keeping the total bandwidth (at least approximately) constant. The waveform dimensionality expansion is particularly achieved by allowing the individual CPMs at individual antennas to have different set of parameters. The expansion in waveform domain allows the diversity code to be fully constrained to the spatial-waveform domains. This leaves the temporal domain intact and available to be used without any constraints by the outer code, e.g. for the coding gain increase.

IV. FULL RANK OPTIMIZED CPFSK WAVEFORMS

A. Optimization targets

The waveform domain code is obtained by the optimization procedure. We restrict ourselves to the case of the full response CPFSK type of phase functions and a binary channel symbol alphabet. Following our goal of designing an inner MWM-only based diversity adding waveform construction, we limit ourselves to optimizing only the prototype MWM functions with the phase state (\( \psi \)) and temporal index \( n \) stripped away

\[
s_0(c_i, t) = e^{j2\pi c_i \psi(t)} (\Psi(t) - \Psi(t - T_3)).
\]

The optimization target is the full rank and the maximum determinant of the codeword difference Gram matrix of inner products

\[
G_0(\alpha_1, \alpha_2) = \begin{bmatrix}
\langle \Delta s_{01}(t); \Delta s_{01}(t) \rangle & \langle \Delta s_{01}(t); \Delta s_{02}(t) \rangle \\
\langle \Delta s_{02}(t); \Delta s_{01}(t) \rangle & \langle \Delta s_{02}(t); \Delta s_{02}(t) \rangle 
\end{bmatrix}.
\]

The Gram matrix depends on IMC-CPM parameters \( \alpha_k = [\alpha_{k1}, \alpha_{k2}]^T \). Because of the binary alphabet, the signal difference has only one value \( \Delta s_{01} = s_0(1,t) - s_0(2,t) \). The inner product can be calculated either in time domain or using the signal space representation. We used the former case. Notice that the Gram matrix for full MWM functions \( s_{i,c_{n,i},t} \) is

\[
G = G_0 \odot F
\]

where \( \odot \) is Hadamard product and elements of the matrix \( F \) are \( F_{ij} = e^{j \psi(i \cdot - j \cdot \cdot \cdot )} \).

The optimization yields optimal values of \( \alpha_k \) in (3). We assume \( \kappa_1 = \kappa = 1/2 \) for the compatibility with the reference scalar MSK modulation. Notice that the effective modulation index of IMC-CPFSK cannot exceed this value. This guarantees (at least approximately) that the bandwidth of the signal will not be increased.
approximately bandwidth (c) design object given by waveform shape requirements given ad-hoc (a) temporal domain waveform domain (b) temporal domain waveform domain Tx1 and Tx2 codeword (spatial domain) codeword with symbols with unconstrained connections codeword with symbols with constrained connections waveform pulse basis function

Figure 1. Examples of the space-time-waveform codeword construction. (a) Traditional temporal domain Alamouti code over 1-dimensional (linear) modulation. (b) Waveform domain Alamouti code (e.g. over OFDM sub-carriers). (c) Space-Waveform code with increased dimensionality, optimized waveforms and waveform shape constraints.

B. Optimization results

The optimization was carried out by a semi-numerical approach. The details are skipped for brevity. First, the Gram matrix was derived analytically. Subsequently, the rank and the determinant were evaluated. The determinant maximization itself was carried out numerically. In order to lower the dimension of the optimization problem, two options of 2-dimensional (instead of a full 4-dimensional) optimization were set:

(option 1) \[ \alpha_1 = \begin{pmatrix} +1 \\ 1 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{pmatrix}; \] (10)

(option 2) \[ \alpha_1 = \begin{pmatrix} +1 \\ \hat{\alpha}_1 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} -1 \\ \hat{\alpha}_2 \end{pmatrix}. \] (11)

Notice that the option 1 tries to verify whether two CPF-SKs with mutually orthogonal channel waveforms would be optimal. This would be in fact two MSK modulations (for \( \alpha_2 = [-1, +1] \)) on two Tx antennas with mutually swapped (i.e. mutually orthogonal) symbols. Surprisingly, this is not the optimal case (see below).

The numerical optimization is show on Fig. 2. The resulting optimal values maximizing the determinant are

\[ \hat{\alpha}_1 \approx -0.26, \quad \hat{\alpha}_2 \approx 0.26. \] (12)

The rank is equal to 2 (full-rank) for those values. Observing the results, we can draw an important observation. The optimal values does not correspond to the 2-antenna MSK. Most importantly, the 2-antenna MSK waveforms does not have full rank. The rank is 1 for \( \alpha_2 = [-1, +1] \). The resulting optimized waveforms does have (as was conjectured) increased dimensionality \( N_w = 4 \), and the resulting waveforms are nicely mirror-symmetrical. The dimensionality expansion is needed in order to obtain the full diversity, even at the price of losing mutual orthogonality of waveforms at different antennas.

C. Spatially Mirrored Tilted Phase CPFSK

The optimization results into phase tree on Tx1 and Tx2 with tilted and mutually mirrored trajectories (Fig. 3). This immediately leads to an elegant implementation with Spatially Mirrored Tilted Phase (SMTP) CPFSK (Fig. 4). The signal on Tx1 and Tx2 is frequency tilted and spatially mirrored

\[ s_1(t) = s_0(t) e^{j2\pi \hat{f}t}, \]
\[ s_2(t) = s_1^*(t) = s_0(t) e^{-j2\pi \hat{f}t}, \] (13) (14)

where

\[ s_0(t) = \exp \left( j2\pi \kappa \sum_n c_n \beta_0(t - nT_3) \right). \] (15)
κ is an ordinary CPFSK signal with effective modulation index κ_e and symbols c_n ∈ {±1}.

The optimization maximum is relatively flat and we can, in order to facilitate the CPFSK implementation (finite number of states), further approximate the optimal values by \( \hat{\alpha}_1 = -\alpha, \hat{\alpha}_2 = \alpha, \alpha \approx 1/3 \). The effective modulation index can be get by \( \kappa(1 + \hat{\alpha}) = 2\kappa_e \Rightarrow \kappa_e = 1/3 \). The corresponding CPE will have \( M_s = 6 \) states. The frequency tilt can be get by

\[
2\pi f_{TS} = \pi/2 - 2\pi/6 \Rightarrow \hat{f} = 1/(12T_S).
\]

D. BER performance verification

The BER performance of SMTP-CPFSK system defined by Fig. 4 was verified by means of a computer simulation and also by a semi-numerical evaluation of the Pairwise Error Probability (PEP). The results are in Fig. 5. In the first approach, we only evaluated the performance of the MWM constellation signaling, i.e. with the CPE memory ignored. The CPE itself should not affect the diversity gain only the coding gain. It has a similar effect as any other outer code with a memory. Our goal was to observe the \(-2\) BER curve slope proving the full rank diversity in 2 Tx antenna system. The results show that this is truly achieved. We also included the reference scalar MSK system. Both systems occupy approximately the same bandwidth.

V. DISCUSSION OF RESULTS AND CONCLUSIONS

The paper presented a design of the space-waveform diversity code for the nonlinear CPM modulation in MIMO channel. In the special case of binary alphabet, 2 Tx antennas and full response CPFSK, the rank and determinant optimization leads to an elegant implementation with Spatially Mirrored Tilted Phase (SMTP) CPFSK. The signals on Tx1 and Tx2 are frequency tilted and mutually spatially mirrored. The resulting scheme has the full rank diversity which is achieved purely in the waveform domain with the help of its dimensionality expansion.

REFERENCES