Network Coded Modulation for Random Channel Class in WNC with HDF Relaying Strategy

Jan Sykora Alister Burr

Abstract—**A standard NCM (Network Coded Modulation) for WNC (Wireless Network Coding) with HDF (Hierarchical Decode & Forward) strategy requires that each node has full knowledge of the connectivity pattern to other nodes and this knowledge affects the design of NCM and HNC (Hierarchical Network Code) maps at each transmitting (source) and receiving (relay) node respectively. The final destination has to have full knowledge of all HNC maps used and the composite overall HNC map needs to be invertible to recover the source data. This approach collapses if nodes do not have a full knowledge of the connectivity pattern and thus cannot adjust their HNC maps properly. We design the NCM and HNC maps for WNC with HDF strategy that works regardless of this knowledge. It is based on a concept of a *channel class* which is a *discrete* (random) parameter describing the channel behavior and which becomes a part of HNC maps and the NCM hierarchical constellation. We analyze the rate regions for this design and compare it to the standard NCM and HNC map solution suffering from random channel outages.**

Index Terms—**Wireless Network Coding, Random Channel Class, Network Coded Modulation**

I. Introduction and System Model

Background And Related Work: The paradigm of *network coding* (NC) [1] can significantly increase the efficiency of multiuser, multihop networks by allowing multiple data streams to be combined and to "flood" through the network via multiple routes. The final destination must then collect enough related observations to solve for the data it requires. However NC operates only at the network level, and upon discrete symbols. Wireless Network Coding (WNC), a.k.a. Physical-Layer Network Coding, transfers this paradigm into *signal constellation space*, allowing for the unavoidable superposition of radio signals at the receiver antenna, which means that the network code must be designed jointly with the signal constellation. WNC provides a *network structure aware* physical layer operating over the whole network, rather than being restricted to individual links between nodes.

We use here the terminology of [2] to describe the components of our WNC strategy. Network Coded Modulation (NCM) refers to the relationship between the network code symbols and the corresponding constellation-space signals. The network code symbols are related to the source symbols by a mapping function called the Hierarchical Network Code (HNC) map, and we refer to these symbols as hierarchical symbols. In this paper we assume that relays use a Hierarchical Decode and Forward (HDF) strategy, in which hierarchical symbols are directly decoded at the relay and forwarded towards the destination(s). A given destination collects all available observations of hierarchical symbols related to the data of interest to it, plus any Complementary Side Information (C-SI) which may help to solve for this data.

The context of our work includes the following selected references. The first attempts to design proper relay decision maps are described in [3], [4], [5], where the authors use symbol-wise adaptive mappings designed to maximize throughput. The authors of [6] and [7] use constructions based on lattice codes [8], and this approach is generalized in the compute-and-forward strategy of [9], [10], and similarly in [11]. [2] introduces a new layered approach to NCM design.[12], [13] address the effect of channel fading parameters and imperfect C-SI, very particular to WNC. [14] addresses a number of information theoretic results including synchronization and topology related issues.

Goals and Contributions: It would normally be assumed in the design of a WNC strategy that each node has full knowledge of all channels, and especially the connectivity between nodes, so that the HNC maps at each node can be selected to ensure that the composite map is invertible. The object of this paper is to present the concept of a WNC strategy that works *regardless* of connectivity knowledge. We classify a channel in terms of a discrete parameter which defines its *channel class*. In the simplest case a binary channel class indicates connection or disconnection.

The core concept is to treat the channel class as an additional degree of freedom of the received signal, and therefore to treat the index of the channel class in the same way as an information symbol. Hence the overall HNC map must be invertible not only in terms of the data, but also of the class index of all channels. We will also show the important role of the constellation space NCM mapping function.

The object of the paper is to describe this concept rather than to present a complete design theory. For this reason we do not consider an exhaustive set of scenarios, but rather demonstrate the principle using the simplest possible scenario that permits this. However we provide in outline an example showing the usefulness of our approach and pointing out differences from the classical "signaling" solution. The signaling solution (Fig. 1) is based on sensing the channel and then *switching* the NCM map at the node depending on the channel class (connection/disconnection). This sensing and switching

IEEE-L-COM, C5 (20.2.2013)
This work was supported by COST Action IC1004, FP7-ICT SAPHYRE and DIWINE projects, GACR 102/09/1624, and MSMT LD12062.
The authors are with Czech Technical University in Prague (jan.sykora@fel.cvut.cz) and the University of York (agb1@ohm.york.ac.uk).

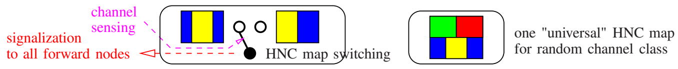

Figure 1. HNC map switching for classical signaling solution and "universal" HNC map for random channel class.

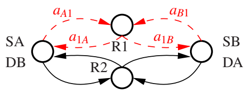

Figure 2. 2-Way 2-Relay system example with random unreliable links (red, dashed) associated to relay R1.

is relatively easily solved for links ending at the node where the map should be switched. It can be combined with channel estimation using a suitable preamble, which would be required anyway. Much more important and harder to avoid is that this map must be made available to all succeeding nodes and stages of the larger multi-hop network. This creates at least a *latency*. Since the signaling information must be forwarded reliably it must be coded by long codewords (even if it is itself very short). This cannot be done at the channel symbol level. Scenarios with multiple hops, connectivity loops, and short packet services with rapidly changing channel classes are particularly vulnerable to this. In some cases this reliable signaling is not possible at all. In Fig. 2, all links at R1 have random channel classes and the destination has no reliable way to obtain the HNC map used at R1. Our approach with HNC map inherently containing the channel class does not need this explicit signaling. Of course it is at the expense of larger output cardinality. But since the bottleneck is usually the MAC stage, this may not restrict the overall capacity. Moreover, the channel class is contained in each payload channel symbol. This works like a long "repetition" code, so that a long channel class observation is spread over the whole payload. Note that to demonstrate the generality of our approach, we do not assume that the state of forward and reverse links are correlated.

System Model and Definitions: Sources are denoted Sm, $m \in \{A, B, \ldots\}$, and relays Ri, $i \in \{1, 2, \ldots\}$. Each source transmits constellation space symbols $s_m(c_m) \in \mathscr{A}_s$ which are one-to-one mappings from the code symbols c_m and are drawn from the common alphabet $\mathscr{A}_s \subset \mathbb{C}^{N_s}$ with cardinality M_s and dimensionality per symbol N_s. The received signal at the relay Ri is $x_i = \sum_m h_{mi}(a_{mi}, \theta_{mi}) s_m(c_m) + w_i$ where h_{mi} are channel transfer coefficients between source node m and relay node i, $a_{mi} \in \mathbb{Z}$ are discrete channel classes, θ_{mi} are continuous valued channel parameterizations (for simplicity these will henceforth be omitted), perfect symbol timing is assumed, and w_i is Gaussian noise with variance σ_w^2 per complex dimension. The channel class constitutes Channel Class Information (CCI).

The channel class a_{mi} is a *discrete valued* parametrization of the channel link. As such, it creates additional discrete valued degrees of freedom in the received signal that is similar to the way in which the discrete code symbols c_m affect the received signal. This is a fundamental observation which motivates the joint data and channel class NHC map design described later in the paper. The channel class itself can describe a variety of channel characteristics (e.g. switching between various stochastic characteristics of the fading). However the most important and the simplest interpretation for the channel class is binary modeling of connectivity (link present or absent). The connectivity can be lost due to fading.

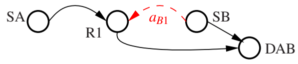

Figure 3. 2-Source 1-Relay system example.

The 2-source 1-relay (2S-1R) system (Fig. 3) is the simplest possible example that is capable of demonstrating the functionality of NCM/HNC in the scenario with a random channel class. It is in fact an asymmetric MAC-relay channel. The system has only one channel class variable $a_{B1} \in \{0, 1\}$ which models switching on-off the SB-R1 connectivity. Although there are four links we assume variable connectivity at one link only as the simplest case. The relay then processes a hierarchical function of the source symbols, including the CCI of links affecting the relay. The destination has this function available, and also pure C-SI from SB.

II. NCM & HNC-MAPS FOR RANDOM CHANNEL CLASS

In general multi-source multi-relay systems, individual relays process the received signal and make the decisions not on individual data but on some function of data from nodes SA, SB. This is called a hierarchical code symbol (see [2]) and its dependance on source symbols is defined by the HNC map. The overall combined HNC map (including both data and CCI) will be termed the Hierarchical Network Code and Channel (HNCC) map and it defines the contents that must be available at the final destination to recover both the data and the channel classes. Each relay processes its own subset $c_{ABi} = \chi_i(c_A, c_B, \mathbf{a})$ and this information is re-sent towards the destinations.

The final destination receives a set of constellation space signals that provide the decision statistic for a union of HNCC maps $c_{AB} = [c_{AB1}, c_{AB2}]^T$ which is again a function of source code symbols and CCI $c_{AB} = \chi(c_A, c_B, \mathbf{a})$. The vector of the CCI $\mathbf{a} = [a_{A1}, a_{B1}, a_{A2}, a_{B2}]^T$ contains all channel classes of all links from sources to relays. All code and channel class variables are defined as vectors on the extended finite (typically binary) Galois Field (GF) $\mathbb{F}_{2^n}$. The HNCC mapping function $\chi(\cdot)$ must be invertible in order to recover data at the destination. It is in general a nonlinear function, but a number of convenient simplifications become available if it is linear. In this case the HNCC is described by the matrix multiplication $c_{AB} = \mathbf{X}[c_A, c_B, \mathbf{a}]^T$. The element $[\mathbf{X}]_{kj}$ describes the GF coefficient of the j-th source variable (the j-th element of $[c_A, c_B, \mathbf{a}]^T$) in the k-th variable of c_{AB}.

The HNCC map defines how a hierarchical symbol relates to the original source symbols, while the hierarchical constellation defines how it is represented as a wireless signal. The hierarchical constellation $\mathscr{A}_u$ is the useful (noiseless) part of

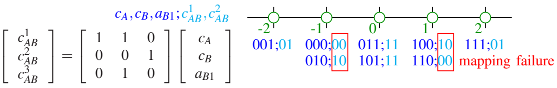

Figure 4. NCM hierarchical constellation at the relay for 2S-1R option #1.

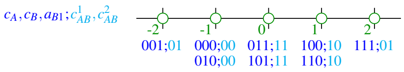

Figure 5. NCM hierarchical constellation at the relay for 2S-1R option #2.

the signal constellation at the relay viewed as a function of the hierarchical symbol $u_i(c_{ABi}) \in \mathscr{A}_u \subset \mathbb{C}^{N_s}$. The overall mapping between source and code symbols (extended in our case to include also channel classes) and hierarchical symbols is called Network Coded Modulation (NCM) [2] $c_A, c_B, \mathbf{a} \mapsto u_i(c_{ABi})$. The design of the NCM, including both the hierarchical constellation $\mathscr{A}_u$ and the corresponding HNCC map χ_i, is thus a critical procedure requiring careful optimization. In the following, we demonstrate the construction of the HNCC map and the corresponding NCM, and also illustrate the concepts described above, in the 2S-1R system example.

HNCC map — option #1: The source and hierarchical alphabet is binary $c_m, c_{AB}^k \in \{0,1\}$. The CCI vector $\mathbf{a} = [a_{B1}]$ contains only a single element and the channel class is binary $a_{B1} \in \{0,1\}$ with the corresponding channel transfer $h_{B1}(a_{B1}) = a_{B1}$. The constellation is BPSK $\mathscr{A}_s = \{\pm 1\}$, $N_s = 1$, $M_s = 2$, $s_m(0) = -1$, $s_m(1) = +1$. The *linear* HNCC map is shown in Fig. 4 and its HNCC matrix $\mathbf{X}$ is a full rank matrix *invertible* over the GF. Hierarchical 4-ary symbols $c_{AB1} = [c_{AB}^1, c_{AB}^2]^T$ are processed by the relay and the symbol $c_{AB2} = [c_{AB}^3]$ is directly available to the destination by the side-link (dummy relay).

The hierarchical constellation (for the combined code and channel class) is $\mathscr{A}_u = \{-2,-1,0,1,2\}$ and its mapping onto the hierarchical code symbols and corresponding source data and channel classes is shown in Fig. 4. We show only the hierarchical constellation at the relay. The decision metric for full HNCC map at the destination is a union of that obtained from the relay and the side-link. The complementary side-information (SB-DAB) is assumed to be perfect (see [13], [15] for the imperfect case).

It is apparent that this particular NCM exhibits critical *mapping failures* at which two distinct hierarchical code symbols $c_{AB1} = [c_{AB}^1, c_{AB}^2]^T$ correspond to one constellation point. This mapping failure makes it impossible to distinguish the source code symbol c_A if $a_{B1} = 0$. Notice that this failure is specific to the constellation space hierarchical map (NCM) even if the underlying HNCC map is fully invertible, and that the phenomenon occurs only for a specific value of the channel class. This justifies the inclusion of the channel class into the HNCC map in order to be able to identify such a case.

HNC map — option #2: In order to avoid hierarchical constellation failure we need to use a hand-crafted *nonlinear* mapping as shown in Fig. 5.

III. Hierarchical Rates in MAC Stage

We evaluate mutual information rates which in fact are cut-set bounds on the rates of the hierarchical symbol c_{AB1}: $C_{AB1} = I(c_{AB1}; x_1), C_{AB1}(a) = I(c_{AB1}; x_1 | a_{B1}), C_{AB1}(c) = I(c_{AB1}; x_1 | c_A, c_B)$ where x_1 is the received constellation space symbol at relay R1. These upper bound rates are *not* affected by the channel class, and should be interpreted as follows. The rate C_{AB1} describes the information rate carried by the received hierarchical symbol c_{AB1} when both the source symbolsc_A, c_B and the channel class a_{B1} are unknown. In simple terms, it means how much information in total is carried in the received signal x_1 about c_A, c_B and a_{B1} jointly represented by the function $c_{AB1} = \chi_1(c_A, c_B, a_{B1})$. The rates $C_{AB1}(a)$ and $C_{AB1}(c)$ refer to the information carried when only c_A, c_B, and only a_{B1} are unknown, respectively: that is, they are the information in x_1 respectively about the source symbols (taken jointly) and about the channel class. With the assumption of a *perfect* side-link, it means for source A rate R_A and the channel class rate (given by the class cardinality) R_{CCI}: $R_A + R_{\mathrm{CCI}} < C_{AB1}$, $R_A < C_{AB1}(a)$, and $R_{\mathrm{CCI}} < C_{AB1}(c)$. Please compare this with the classical map switching where the *particular* value of $a_{B1}$is known via a separate reliable signaling channel and the mutual information is $I(c_{AB1}; x_1 | a_{B1} = 0), I(c_{AB1}; x_1 | a_{B1} = 1)$.

The *reference* case for comparison is a NCM/HNC map designed as if the channel class did not change. In our 2S-1R example it means assuming $a_{B1} = 1$ when designing NCM/HNC. Then the channel SB-R1 suffers from channel outage. If an outage occurs the information rate of this scheme degrades. The mean hierarchical rate of this reference case is denoted by $\mathrm{E}[C_{AB1r}]$. We also evaluate the value which C_{AB1r} attains when there is no channel outage, i.e. the channel class is fixed $a_{B1} = 1$. This is the standard situation for NCM with fixed channel class. Finally, we show the single user Gaussian alphabet capacity. The Signal-to-Noise Ratio at the relay is $\gamma_x = \mathrm{E}\left[\|s_A\|^2\right] / \sigma_w^2$. Rate evaluation uses the technique of [2].

Results for the 2S-1R scenario with BPSK alphabet and HNCC options #1 and #2 are given in Figs. 6 and 7. The channel has a constant transfer coefficient with no continuously variable parametrization (for simplicity) $h_{A1} = 1$, $h_{B1}(a_{B1}) = a_{B1}$, $a_{B1} \in \{0,1\}$ with $\Pr\{a_{B1} = 1\} = 0.5$. We can clearly see the penalty paid in option #1 for mapping failures. All hierarchical rates C_{AB1} are substantially lowered in comparison with option #2. The asymptotic bound on C_{AB1} reaches only 1.5 bits which nicely corresponds to the fact that the source A symbols cannot be distinguished when the channel class is $a_B = 0$ and this happens with probability $\Pr\{a_B = 1\} = 0.5$. Option #2 does not suffer from this phenomenon. This justifies us to consider further only option #2.

We can also see that all rates forming the cut-set bound for combined data-channel hierarchical symbols, i.e. $C_{AB1}, C_{AB1}(a), C_{AB1}(c)$, are better than the mean rate $\mathrm{E}[C_{AB1r}]$ of the reference NCM scheme designed as if the channel class was always $a_{B1} = 1$. This result fully justifies our design approach. It is worth noting that the the capability of NCM with an HNCC map is paid for by the necessity of having

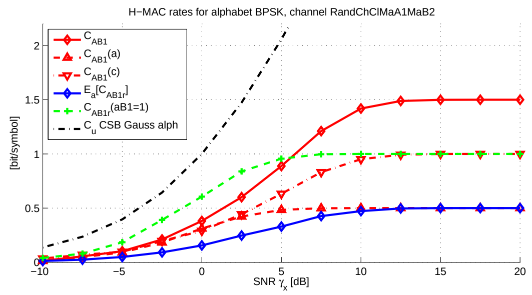

Figure 6. A comparison of rates for 2S-1R HNCC map option #1.

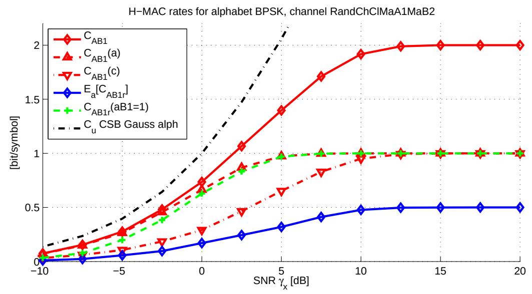

Figure 7. A comparison of rates for 2S-1R HNCC map option #2.

higher cardinality at the output of the relay. This is not usually a problem since the performance bottleneck lies in the MAC stage, not in the BC stage. Also, the redundant NCM/HNCC, which is capable of working regardless of the channel class, is worth using only if the channel class really is randomly variable. If we were sure that there was only one channel class $a_{B1} = 1$, then the standard NCM/HNC (Fig. 7, $C_{AB1r}(a_{B1} = 1)$ curve) in the MAC stage, would require lower cardinality relay output at the BC stage, while having a comparable performance. However it should be stressed that, if the channel class remains constant over a longer frame, the cardinality increase is diluted over this frame since only one channel class variable per frame is needed. In particular it means that in the cut-set bound region (R_A, R_{CCl}) the position of the operation point will be strongly biased towards the R_A axis. Therefore, asymptotically for a very long frame with common channel class the scheme approaches the $R_A < C_{AB1}(a)$ performance. We can also notice (Fig. 7) that the rate $C_{AB1}(a)$ is marginally better than $C_{AB1r}(a_{B1} = 1)$ at low SNR. It appears to be caused by the fact that the rate $C_{AB1}(a)$ represents capabilities of two constellations, for $a_{B1} = 0$ and $a_{B1} = 1$. The better "quality" of the constellation $\{\pm 1\}$ for $a_{B1} = 0$ improves the rate.

IV. Discussion and Conclusions

We have developed a proper system model allowing the design of NCM for a random channel class. An important special case of the channel class is the binary case which models the *connectivity* pattern. The core idea of this paper is based on the fact that the discrete channel class must be treated in the NCM and HNCC map design as an entity equivalent to the information carrying data. This allows us to systematically design an NCM/HNCC that works *regardless* of the channel state. We may compare this to the standard solution where NCM/HNC is designed for one channel class and we only passively analyze what happens when the channel class varies randomly. The second major achievement of the paper is the observation that the design of NCM and HNCC must be jointly optimized. It is *not enough* to have a properly invertible combined data and channel class HNCC map. The corresponding combined (data and channel class) hierarchical constellation might demonstrate mapping failures strongly affecting the data rates. The optimal solution (2S-1R binary example) might not even be a *linear* HNCC map. Numerical results evaluating the mutual information of hierarchical HNCC rates justify the superiority of our NCM/HNCC design for randomly variable channel classes in comparison with a standard NCM/HNCC solution which suffers with the performance degradation for random channel classes.

References

[1] R. W. Yeung, S.-Y. R. Li, N. Cai, and Z. Zhang, *Network Coding Theory*. now Publishers, 2006.
[2] J. Sykora and A. Burr, "Layered design of hierarchical exclusive codebook and its capacity regions for HDF strategy in parametric wireless 2-WRC," *IEEE Trans. Veh. Technol.*, vol. 60, pp. 3241–3252, Sept. 2011.
[3] P. Popovski and T. Koike-Akino, "Coded bidirectional relaying in wireless networks," in *Advances in Wireless Communications* (V. Tarokh, ed.), Springer, 2009.
[4] T. Koike-Akino, P. Popovski, and V. Tarokh, "Denoising strategy for convolutionally-coded bidirectional relaying," in *Proc. IEEE Internat. Conf. on Commun. (ICC)*, 2009.
[5] T. Koike-Akino, P. Popovski, and V. Tarokh, "Optimized constellations for two-way wireless relaying with physical network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, pp. 773–787, June 2009.
[6] W. Nam, S.-Y. Chung, and Y. H. Lee, "Capacity bounds for two-way relay channels," in *Proc. Int. Zurich Seminar on Communications*, 2008.
[7] I.-J. Baik and S.-Y. Chung, "Network coding for two-way relay channels using lattices," in *Proc. IEEE Internat. Conf. on Commun. (ICC)*, 2008.
[8] U. Erez and R. Zamir, "Achieving $1/2\log(1+\text{SNR})$ on the AWGN channel with lattice encoding and decoding," *IEEE Trans. Inf. Theory*, vol. 50, pp. 2293–2314, Oct. 2004.
[9] B. Nazer and M. Gastpar, "Computation over multiple-access channels," *IEEE Trans. Inf. Theory*, vol. 53, pp. 3498–3516, Oct. 2007.
[10] B. Nazer and M. Gastpar, "Compute-and-forward: Harnessing interference through structured codes," *IEEE Trans. Inf. Theory*, vol. 57, pp. 6463–6486, oct 2011.
[11] M. P. Wilson, K. Narayanan, H. D. Pfister, and A. Sprintson, "Join physical layer coding and network coding for bidirectional relaying," *IEEE Trans. Inf. Theory*, vol. 56, pp. 5641–5654, Nov. 2010.
[12] T. Uricar and J. Sykora, "Design criteria for hierarchical exclusive code with parameter-invariant decision regions for wireless 2-way relay channel," *EURASIP J. on Wireless Comm. and Netw.*, vol. 2010, pp. 1–14, July 2010.
[13] J. Sykora and A. Burr, "Network coded modulation with partial side-information and hierarchical decode and forward relay sharing in multi-source wireless network," in *Proc. European Wireless Conf. (EW)*, (Lucca, Italy), pp. 1–7, Apr. 2010.
[14] S. C. Liew, S. Zhang, and L. Lu, "Physical-layer network coding: Tutorial, survey, and beyond," *Physical Communications*, no. doi:10.1016/j.phycom.2012.05.002, pp. 1–39, 2012.
[15] A. Burr and J. Sykora, "Extended mappings for wireless network coded butterfly network," in *Proc. European Wireless Conf. (EW)*, (Vienna, Austria), pp. 1–7, Apr. 2011.