Determinant Criterion Optimizing Linear Subspace Projector for Burst Orthogonal STC CPM Modulation in MIMO Channel

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***Abstract*—We consider a general burst block orthogonal space-time coded CPM modulation in MIMO slowly fading Rayleigh channel. The modulated signal has multidimensional constellation waveforms. The optimal receiver must perform a multidimensional matched filtering on its front-end. This operation is highly demanding on the processing computational resources. We derive sub-optimal complexity reducing receiver linear preprocessor which reduces the dimensionality of the signal. We analytically derive the optimal linear projector matrix for several cases of the free trellis path and its termination. The determinant optimization is shown to lead to an eigenvector problem solution. It is a two-fold joint optimization problem with 2 mutually coupled domains (similar to a decoding in a parametric channel) with number of solution options with generally high complexity. We adopt simplified approach decoupling this 2 domains. As an example, we apply the procedure for a particular case of one-dimensional subspace projector and Burst Alamouti coded MSK modulation as a special case of STC CPM modulation.**

I. Introduction

A. Background

Space-Time Coding (STC) over linear modulations in MIMO (Multiple-Input Multiple-Output) channel is well covered topic with huge number of results (see e.g. [1] and the references therein). Unlike this, the *nonlinear* space-time coded modulation in MIMO environment, due its inherent complexity, remains explored only sparsely. Several results on coding schemes, detection, synchronization and capacity can be found in [2], [3], [4], [5], [6], [7], [8]. The nonlinear modulation are mainly attractive due to their ability to decouple 2nd order signal properties from the waveform shape. The best example is the class of Continuous Phase Modulation (CPM) with a constant envelope property. This makes the signal resistant to a nonlinear distortion, e.g. by the power amplifier.

B. Goals

We derive a closed-form solution of the optimal *linear waveform subspace projector* for burst Orthogonal Block (OB) space-time coded CPM signal. We utilize a special structure of the code that allows to get a simple analytical solution for the optimal projection basis. The projection basis is required to be *orthogonal* and *Nyquist* in order to facilitate the receiver metric computation (Laurent expansion [9] including its various tilted forms does not satisfy this). The procedure is exemplified on the simplest possible scenario with the MSK (Minimum Shift Keying) modulation as a special case of CPM and Alamouti code in the 2×1 MIMO system. The procedure itself has a general validity.

C. Prior Art And Related Work

A numerical ad-hoc optimization of the special *linear* projector for block burst Alamouti space-time coded CPM was treated in [10]. The procedure was based on earlier work defining the optimality criterion in [11]. The problem of the dimensionality reducing complexity reduction performed by a preprocessor can be also approached from the different perspective—by the nonlinear preprocessing. This approach is investigated in [12], [13].

II. System Model and Definitions

Our general target system consists of space-time coded CPM with concatenated block-wise orthogonal design spatial code.

A. Waveform Space Representation of CPM Modulation

Each antenna transmits space-time coded CPM modulation $s(t) = \sum_n g_n(t - nT_S, q_n, \sigma_n)$. The nonlinear modulation pulse is

$$g_n(t, q_n, \sigma_n) = e^{j\left(\sigma_{n,L} + 2\pi\kappa\sum_{\ell=1}^{L-1}\sigma_{n,\ell}\beta(t+\ell T_S) + 2\pi\kappa q_n \beta(t)\right)} v_1(t) \quad (1)$$

where $\beta(.)$ is the phase shaping function with the correlation length L, $v_1(t) = (\mathscr{U}(t) - \mathscr{U}(t - nT_S))$ is a Nyquist masking pulse, $\mathscr{U}(.)$ is the Unit Step function, κ is a modulation index, T_S is the symbol duration, $q_n \in \{\pm 1\}$ are binary channel symbols (generalization for an arbitrary alphabet is straightforward). The states are governed by

$$\sigma_{n+1,1} = q_n,\ \sigma_{n+1,2} = \sigma_{n,1}, \ldots, \sigma_{n+1,L-1} = \sigma_{n,L-2},$$
$$\sigma_{n+1,L} = (\sigma_{n,L} + \pi\kappa\sigma_{n,L-1}) \bmod 2\pi. \quad (2)$$

This is a traditional canonic form. The second option is to use Laurent expansion [9]. This is in fact a representation of the

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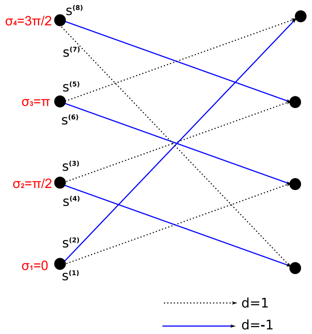

Figure 1. MSK trellis. The modulator states are $\sigma_n \in \{0,\ \pi/2,\ \pi,\ 3\pi/2\}$.

CPM using a linear multidimensional expansion of the signal corresponding to one symbol. However a major problem of Laurent expansion is the fact that it is *not* complying with the *Nyquist criterion*. The (pseudo)pulses are non-orthogonal. The failure of being a Nyquist one makes problems in designing and analyzing a code concatenated with the CPM. For that reason, we use a Nyquist waveform signal space representation.

The waveform space basis must be generally found by the Gram-Schmidt procedure. Arbitrary CPM pulse $g(.)$ with the phase correlation length $L = 1$ has the basis with $N_s = 2$ dimensions (complex).

In a *particular case of MSK*, and has a form

$$\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix},\ \mathbf{A} = \begin{bmatrix} 1 & 0 \\ j\frac{a_1}{a_2} & \frac{1}{a_2} \end{bmatrix}, \tag{3}$$

where $\{\xi_1(t),\ \xi_2(t)\} = \left\{\exp\left(j\pi\frac{t}{2T_S}\right),\ \exp\left(-j\pi\frac{t}{2T_S}\right)\right\}$ for $t \in [0, T_S)$ and $a_1 = 2/\pi$ and $a_2 = \sqrt{1 - 4/\pi^2}$. The signal space points are drawn from the set

$$\begin{aligned} \mathbf{s} \in \mathscr{A}_s = & \left\{\mathbf{s}^{(i)}\right\}_{i=1}^{8} \\ = & \{(1,0)^T,\ (-ja_1, a_2)^T,\ (j,0)^T,\ (a_1, ja_2)^T, \\ & (-1,0)^T,\ (ja_1, -a_2)^T,\ (-j,0)^T,\ (-a_1, -ja_2)^T\}. \end{aligned} \tag{4}$$

In our notation, symbols with an odd superscript ($\mathbf{s}^{(1)}$, $\mathbf{s}^{(3)}$, ...) are mapped to the data symbol $+1$ and similarly the symbols with an even superscript ($\mathbf{s}^{(2)}$, $\mathbf{s}^{(4)}$, ...) correspond to the data symbol -1. The symbol set exhibits a rotational symmetry $\mathbf{s}^{(3)} = j\mathbf{s}^{(1)}$, $\mathbf{s}^{(5)} = j\mathbf{s}^{(3)}$, $\mathbf{s}^{(7)} = j\mathbf{s}^{(5)}$, $\mathbf{s}^{(1)} = j\mathbf{s}^{(7)}$, $\mathbf{s}^{(4)} = j\mathbf{s}^{(2)}$, $\mathbf{s}^{(6)} = j\mathbf{s}^{(4)}$, $\mathbf{s}^{(8)} = j\mathbf{s}^{(6)}$, $\mathbf{s}^{(2)} = j\mathbf{s}^{(8)}$. The modulation memory is described by a trellis (see Fig. 1).

B. Burst Orthogonal Block Code

We assume block-wise burst orthogonal code build from the segments of the CPM modulated signal having total length K (even). As an example, Alamouti code has the time-space matrix

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_{K/2+1} \\ \vdots & \vdots \\ \mathbf{s}_{K/2} & \mathbf{s}_K \\ -\mathbf{s}^*_{K/2+1} & \mathbf{s}^*_1 \\ \vdots & \vdots \\ -\mathbf{s}^*_K & \mathbf{s}^*_{K/2} \end{bmatrix}. \tag{5}$$

The concatenation of signal segments in the code can generally violate the signal continuity otherwise being an essential characteristics of the CPM based system. An impact, mainly on the spectral properties, of this phenomenon will be negligible for large K. But since the large block length would cause unwanted processing delays, we try to address this issue also for relatively short blocks. It would require to control the initial and terminating states of the CPM trellis in order to guarantee the continuity of the phase.

C. Waveform Space Channel Model

The received signal for n-th symbol and k-th receive antenna (dimensionality N_s per antenna) in a signal space notation is

$$\mathbf{x}_{n,k} = \sum_{i=1}^{N_T} h_{ki}\mathbf{s}_{n,i} + \mathbf{w}_{n,k} \tag{6}$$

where h_{ki} are flat fading MIMO channel coefficients and $\mathbf{w}_{n,k}$ are Gaussian noise vectors. This can be written in a compact form using space-stacked vectors and matrices $\tilde{\mathbf{x}}_n = \tilde{\mathbf{H}}\tilde{\mathbf{s}}_n + \tilde{\mathbf{w}}_n$ where $\tilde{\mathbf{x}}_n = [\mathbf{x}^T_{n,1}, \ldots, \mathbf{x}^T_{n,N_R}]^T$ and similarly for $\tilde{\mathbf{s}}_n$ and $\tilde{\mathbf{w}}_n$. The stacked channel matrix has a *Kronecker* structure $\tilde{\mathbf{H}} = \mathbf{H} \otimes \mathbf{I}_{N_s}$ ($\otimes$ is a Kronecker product).

III. Projector Optimality Criterion

A general treatment of the receiver preprocessing options based on a linear projector can be found in [11]. The projector can operate over arbitrary subspace in the Cartesian product of the time domain, the spatial domain and the waveform space domain. Here, we focus only on a special case of the *waveform space projector*.

A. Receiver Front-End Projector

The signal of each receive antenna passes through a linear dimensionality reducing projector. The signal at the projector output is generally $\tilde{\mathbf{y}} = \tilde{\mathbf{P}}\tilde{\mathbf{x}}$ where vectors $\tilde{\mathbf{y}}, \tilde{\mathbf{x}}$ are stacked vectors in the antenna and waveform space and $\tilde{\mathbf{P}}$ is a general projector. In a special case of *waveform-only projector*, it was shown in [11] and [10] that the system with the projector can be modeled by equivalent projected transmitted constellation points $\mathbf{u}_{n,i} = \mathbf{P}\mathbf{s}_{n,i}$ where $\mathbf{P}$ is the common waveform space projection $N_s \times N_s$ matrix with the rank $N_P < N_s$, where N_P is the dimensionality of the projection subspace. For the notation simplicity reasons, we drop the sequential and the antenna indices further on, and we simply write

$$\mathbf{u} = \mathbf{P}\mathbf{s} \tag{7}$$

for a *general projected equivalent symbol*.

The basic properties of linear projectors are [14] $\mathbf{P} = \mathbf{P}^H$, $\mathbf{P}^2 = \mathbf{P}$, $||\mathbf{P}||_F^2 = N_P$, $\det \mathbf{P} = 0$, $\mathbf{PB} = \mathbf{B}$ and $\mathbf{P} = \mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H$, where $(\cdot)^H$ denote Hermitian transposition, and $||\cdot||_F^2$ denotes Frobenius norm. The columns of the matrix $\mathbf{B}$ forms the basis of the projection subspace. Assuming orthonormality of the basis $\mathbf{B}^H\mathbf{B} = \mathbf{I}$, the projector becomes $\mathbf{P} = \mathbf{B}\mathbf{B}^H$.

In a special case of MSK the projector, the matrix $\mathbf{P}$ is a complex (2×2) matrix. The only feasible dimensionality reduction is thus the projection into 1-dimensional $N_P = 1$ space. Then $\text{rank}(\mathbf{P}) = N_P = 1$ and matrix $\mathbf{B}$ becomes a column vector $\mathbf{B} = \mathbf{b}$.

Note that for (2×2) projection matrix

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix}, \tag{8}$$

it can be fully described by two numbers p_1 and p_2, because $p_3 = p_2^*$ (see general properties of linear projectors) and $p_4 = 1 - p_1$ as a result of $||\mathbf{P}||_F^2 = 1 = \text{tr}(\mathbf{P}^H\mathbf{P}) = \text{tr}(\mathbf{P})$, $\text{tr}(.)$ denotes a trace of the matrix.

B. Rank and Determinant Criterion

We adopt a standard code performance criterion for Rayleigh MIMO channel—the rank and the determinant of the inner product (Gram) matrix for codeword pairs differences [1]. Gram matrix of codeword pairs differences for original and projected constellations are $\mathbf{R_s} = \Delta\mathbf{S}^H\Delta\mathbf{S}$, $\mathbf{R_u} = \Delta\mathbf{U}^H\Delta\mathbf{U}$ where codeword (with length K) differences are $\Delta\mathbf{S} = \mathbf{S}^{(a)} - \mathbf{S}^{(b)}$,

$$\mathbf{S}^{(a)} = \begin{bmatrix} \mathbf{s}_{1,1} & \cdots & \mathbf{s}_{1,N_T} \\ \vdots & \ddots & \vdots \\ \mathbf{s}_{K,1} & \cdots & \mathbf{s}_{K,N_T} \end{bmatrix}. \tag{9}$$

The Matrix $\Delta\mathbf{U}$ is defined similarly.

Due to a specific structure of the block burst orthogonal STC-CPM codeword, we have (Alamouti example)

$$\Delta\mathbf{U} = \begin{bmatrix} \mathbf{P}\Delta\mathbf{s}_1 & \mathbf{P}\Delta\mathbf{s}_{K/2+1} \\ \vdots & \vdots \\ \mathbf{P}\Delta\mathbf{s}_{K/2} & \mathbf{P}\Delta\mathbf{s}_K \\ -\mathbf{P}^*\Delta\mathbf{s}_{K/2+1}^* & \mathbf{P}^*\Delta\mathbf{s}_1^* \\ \vdots & \vdots \\ -\mathbf{P}^*\Delta\mathbf{s}_K^* & \mathbf{P}^*\Delta\mathbf{s}_{K/2}^* \end{bmatrix} \tag{10}$$

and the matrix $\mathbf{R_u}$ has a following form

$$\begin{aligned} \mathbf{R_u} &= \begin{bmatrix} \sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2 & 0 \\ 0 & \sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2 \end{bmatrix} \\ &= \sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2 \mathbf{I}_{N_T}. \end{aligned} \tag{11}$$

The matrix has all mutually equivalent eigenvalues $\lambda_j = \lambda = \sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2$, for all $j \in \{1,2,\ldots,K\}$. The determinant is

$$\det \mathbf{R_u} = \left(\sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2\right)^{N_T}. \tag{12}$$

We denote a sequence of symbol differences as $\Delta\mathbf{s} = [\Delta\mathbf{s}_1,\ldots,\Delta\mathbf{s}_K]^T$. All these results hold also for *arbitrary orthogonal burst block code*.

IV. Synthesis of the Optimal Projector

A. Optimal Projector

Now we derive the optimal subspace projection in accordance with the rank and the determinant criterion. Clearly, any nonzero projector applied on arbitrary orthogonal design code *preserves the rank* $\text{rank}(\mathbf{R_s}) = \text{rank}(\mathbf{R_u}) = r$. The determinant criterion should now provide the optimal projector $\hat{\mathbf{P}}$ and its associated basis $\hat{\mathbf{B}}$. The optimization problem is defined as

$$\hat{\mathbf{P}}(\Delta\mathbf{s}) = \arg\max_{\mathbf{P}} \left(\sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2\right)^{N_T} = \arg\max_{\mathbf{P}} \sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2. \tag{13}$$

The optimal projector resulting from the previous maximization can generally depend on particular sequence $\Delta\mathbf{s}$. This means that different sequences will need a different optimal projector. This is practically impossible to implement. In order to obtain one projector valid for arbitrary signal, we constrain the optimization only to the *most vulnerable codeword pair*

$$\hat{\mathbf{P}} = \arg\max_{\mathbf{P}} \min_{\Delta\mathbf{s}} \sum_{n=1}^K ||\mathbf{P}\Delta\mathbf{s}_n||^2. \tag{14}$$

A consequence of this is the fact that the codeword distance spectrum of the projected signal would have a generally *different shape* than the original one (it would not be just shifted). The maximization problem of projected vectors is equivalent to the minimization of the difference between the original and the projected vector, in our notation $\hat{\mathbf{P}} = \arg\min_{\mathbf{P}} \min_{\Delta\mathbf{s}} \sum_{n=1}^K ||\Delta\mathbf{s}_n - \mathbf{P}\Delta\mathbf{s}_n||^2$.

In this optimality problem we first suitably chose the most vulnerable codeword pair by identifying the two distinct trellis paths. This choice depends on a particular trellis of the modulation, on its initial and terminating state (if required) and also on the outer optional concatenated code. Therefore the following treatment distinguishes two different types of burst Alamouti block STC CPM phase termination.

At first, we assume a zero phase initial state and arbitrary termination phase state. It will lead to the only one non-zero difference at the end of the path. However we lose phase continuity between individual bursts. The advantage is that we do not need any termination flush. As a second case, we assume a defined initial and termination state, where the phase continuity is guaranteed.

B. Projection Subspace Basis

From now on, we *restrict* ourselves only on $L = 1$ CPM class (e.g. MSK). The search of the optimal projector will be carried out indirectly by a search of the optimal basis vector $\hat{\mathbf{B}} = \hat{\mathbf{b}}$. This is also a more practical from the implementation point of view since the basis vector can be directly utilized for the definition of the projected point constellation coefficients—the subspace matched filter.

The problem of the basis search is the *constrained optimization problem* ($\mathbf{b}^H\mathbf{b} = 1$ represents unit energy basis)

$$\begin{aligned}\hat{\mathbf{b}} &= \arg \max_{\mathbf{b}:\mathbf{b}^H\mathbf{b}=1} \min_{\Delta \mathbf{s}} \sum_{n=1}^{K} \Delta \mathbf{s}_n^H \mathbf{P}^H \mathbf{P} \Delta \mathbf{s}_n \\ &= \arg \max_{\mathbf{b}:\mathbf{b}^H\mathbf{b}=1} \min_{\Delta \mathbf{s}} \sum_{n=1}^{K} \|\mathbf{b}^H \Delta \mathbf{s}_n\|^2. \end{aligned} \tag{15}$$

C. Simplified Solution of Joint Optimization

The optimization problem above is a general joint optimization w.r.t. vectors $\mathbf{b}$ and $\Delta\mathbf{s}$ that mutually affects each other. We can see a parallel with the joint synchronization and detection problem in the parametric channel. A number of approaches, including true joint optimization and iterative one, are possible. However all of them are complicated. Here, we adopt a simplified first approach with *independent* search over the $\mathbf{b}$ and $\Delta\mathbf{s}$ domains. We will set the $\Delta\mathbf{s}$ as *the most vulnerable (the shortest one)* sequence for *unprojected full-space* case. In other words, the sequence that is the most vulnerable to the detection error in the full-space case is considered as the most vulnerable in the sub-space case as well. This is strictly speaking suboptimal but simple to implement and it has a clear physical interpretation.

D. Rotational Invariance

The optimal projection matrix is invariant to all codeword symbol sequences distinguished by a *common* phase rotation only $\hat{\mathbf{P}}(e^{j\phi}\Delta\mathbf{s}) = \hat{\mathbf{P}}(\Delta\mathbf{s})$. Therefore any two distinct sequences that are minimizing $\sum_{n=1}^{K} \|\mathbf{b}^H \Delta\mathbf{s}_n\|^2$ and are mutually rotated by a common angle share the same optimal projector.

E. Constrained Optimization

The constraint optimization can be solved by the *Lagrange multiplier method*. However, we have to realize that the objective function is a real-valued function of the complex-valued vector $\mathbf{b}$ and, as such, it is not differentiable in a Cauchy-Riemann sense. But only a stationary point is required to be found. We can use a generalized derivative for this task [14], [15]. The generalized derivative obeys the following basic rules

$$\frac{\tilde{\partial} \mathbf{s}^H \mathbf{b}}{\tilde{\partial} \mathbf{b}} = \mathbf{s}^*, \ \frac{\tilde{\partial} \mathbf{b}^H \mathbf{s}}{\tilde{\partial} \mathbf{b}} = \mathbf{0}, \ \frac{\tilde{\partial} \mathbf{b}^H \mathbf{Q} \mathbf{b}}{\tilde{\partial} \mathbf{b}} = (\mathbf{Q}\mathbf{b})^*, \ \mathbf{Q}^H = \mathbf{Q}. \tag{16}$$

The Lagrangian is $\sum_{n=1}^{K} \Delta\mathbf{s}_n^H \mathbf{b}\mathbf{b}^H \Delta\mathbf{s}_n - \lambda \left(\mathbf{b}^H\mathbf{b} - 1\right)$ where $\Delta\mathbf{s}$ is the difference of the most vulnerable codeword pair. A generalized differentiation of the Lagrangian gives $\sum_{n=1}^{K} \Delta\mathbf{s}_n^* \mathbf{b}^H \Delta\mathbf{s}_n = \lambda \mathbf{b}^*$ or equivalently

$$\sum_{n=1}^{K} \mathbf{Q}_n \mathbf{b} = \lambda \mathbf{b} \tag{17}$$

where $\mathbf{Q}_n = \Delta\mathbf{s}_n \Delta\mathbf{s}_n^H$ and we also define $\mathbf{Q} = \sum_{n=1}^{K} \mathbf{Q}_n$. Since $\alpha_n = \Delta\mathbf{s}_n^H \mathbf{b}$ are scalars, we clearly see that the optimal basis is a linear combination of the vectors $\Delta\mathbf{s}_n$ with weights $\mathbf{b}^H \Delta\mathbf{s}_n / \lambda$, where λ is such that the basis has an unit energy. This means that the basis vector lies in the expansion space of $\{\Delta\mathbf{s}_n\}_n$.

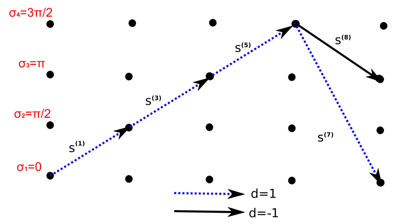

Figure 2. Minimal diverted paths for zero-state start

F. Eigenvector Solution for Optimal Basis

We now easily find a solution for a general case based on (17). The optimal basis must be an eigenvector of (17). Since the stationary point condition does not specify the minimum or maximum optimization goal, we have to select manually the correct eigenvector. However in some special cases, we are able to find the optimal basis by a somewhat simpler method.

G. Unterminated Trellis

The MSK without an outer concatenated code (apart of block Alamouti) has a memory of the length 1. Its *unterminated* trellis has only minimal free paths of the length 1. Considering the case of unterminated trellis, the free path has only one non-zero difference $\Delta\mathbf{s} = (0,\ 0,\ 0,\ \Delta\mathbf{s}_K)$. This is equivalent of having the first nonzero difference and $K = 1$. The optimal basis is a trivial solution of (17)

$$\hat{\mathbf{b}} = \Delta\mathbf{s}_1. \tag{18}$$

We can also find a simple geometric interpretation of this result. The projection square length is maximized when the length is maximized and this happens when the projection basis aligns with the vector being projected.

One of the possible splitting path segments is $\Delta\mathbf{s}_K = \mathbf{s}^{(1)} - \mathbf{s}^{(2)}$. We have to keep in mind that the modulation is nonlinear and therefore we need to investigate all possible path pairs. All other paths with the length 1 has differences being just a phase rotation of the above. Since the projector matrix is invariant to this rotation, we can keep considering only this one. A substitution of this into (18) gives $\hat{\mathbf{b}} = \left[1 + \frac{2j}{\pi}, -\sqrt{1 - 4/\pi^2}\right]^T$ and

$$\hat{\mathbf{P}} = \begin{bmatrix} 1 + \frac{4}{\pi^2} & -\frac{(2j+\pi)\sqrt{\pi^2-4}}{\pi^2} \\ -\frac{(-2j+\pi)\sqrt{\pi^2-4}}{\pi^2} & 1 - \frac{4}{\pi^2} \end{bmatrix}. \tag{19}$$

H. Terminated Trellis

The MSK with terminated trellis has the shortest free path of the length 2. The terminate trellis has final state equal to its initial state in order to keep the phase continuity between Alamouti blocks. The most vulnerable path pairs (without projection) are

$$\begin{aligned}
\Delta\mathbf{s}_a &= (\mathbf{s}^{(1)}-\mathbf{s}^{(2)},\ \mathbf{s}^{(3)}-\mathbf{s}^{(8)},\ 0,0,\ldots),\\
\Delta\mathbf{s}_b &= (\mathbf{s}^{(1)}-\mathbf{s}^{(2)},\ \mathbf{s}^{(4)}-\mathbf{s}^{(7)},\ 0,0,\ldots),\\
\Delta\mathbf{s}_c &= (\mathbf{s}^{(3)}-\mathbf{s}^{(4)},\ \mathbf{s}^{(6)}-\mathbf{s}^{(1)},\ 0,0,\ldots),\\
\Delta\mathbf{s}_d &= (\mathbf{s}^{(3)}-\mathbf{s}^{(4)},\ \mathbf{s}^{(5)}-\mathbf{s}^{(2)},\ 0,0,\ldots),\\
\Delta\mathbf{s}_e &= (\mathbf{s}^{(5)}-\mathbf{s}^{(6)},\ \mathbf{s}^{(7)}-\mathbf{s}^{(4)},\ 0,0,\ldots),\\
\Delta\mathbf{s}_f &= (\mathbf{s}^{(5)}-\mathbf{s}^{(6)},\ \mathbf{s}^{(8)}-\mathbf{s}^{(3)},\ 0,0,\ldots),\\
\Delta\mathbf{s}_g &= (\mathbf{s}^{(8)}-\mathbf{s}^{(7)},\ \mathbf{s}^{(6)}-\mathbf{s}^{(1)},\ 0,0,\ldots),\\
\Delta\mathbf{s}_h &= (\mathbf{s}^{(8)}-\mathbf{s}^{(7)},\ \mathbf{s}^{(5)}-\mathbf{s}^{(2)},\ 0,0,\ldots).
\end{aligned} \tag{20}$$

Without sacrificing generality, we assume all nonzero differences at the beginning of the sequence. We use a simplified *independent* search over $\mathbf{b}$ and $\Delta\mathbf{s}$. The sequences have a desirable property that the nonzero parts are only a phase rotations of the one canonic sequence. The projector matrix is again invariant to this rotation and we can use, e.g. $\Delta\mathbf{s}_a$ with $K=2$, for the synthesis of the optimal basis.

In the special case of $K=2$, we can use (apart of the general expression (17)) also an alternative method. It is related to the fact that for $K \leq N_s$ and the matrix $\mathbf{Q}=\sum_{n=1}^{K}\Delta\mathbf{s}_n\Delta\mathbf{s}_n^H$ being a sum of outer products, its eigenvectors are a linear combination of the eigenvectors of $\mathbf{Q}_n=\Delta\mathbf{s}_n\Delta\mathbf{s}_n^H$. Those eigenvectors are in turn equal to $\Delta\mathbf{s}_n$. In our particular case, the basis is

$$\hat{\mathbf{b}}=\Delta\mathbf{s}_1+\Delta\mathbf{s}_2=\mathbf{s}^{(1)}-\mathbf{s}^{(2)}+\ \mathbf{s}^{(3)}-\mathbf{s}^{(8)}. \tag{21}$$

We can easily verify that the $\hat{\mathbf{b}}$ is an eigenvector of (17)

$$\begin{aligned}
&\left(\Delta\mathbf{s}_1\Delta\mathbf{s}_1^H+\Delta\mathbf{s}_2\Delta\mathbf{s}_2^H\right)(\Delta\mathbf{s}_1+\Delta\mathbf{s}_2)=\\
&\quad=\Delta\mathbf{s}_1\Delta\mathbf{s}_1^H(\Delta\mathbf{s}_1+\Delta\mathbf{s}_2)+\Delta\mathbf{s}_2\Delta\mathbf{s}_2^H(\Delta\mathbf{s}_1+\Delta\mathbf{s}_2).
\end{aligned} \tag{22}$$

In our case, it is $\Delta\mathbf{s}_1^H(\Delta\mathbf{s}_1+\Delta\mathbf{s}_2)=\Delta\mathbf{s}_2^H(\Delta\mathbf{s}_1+\Delta\mathbf{s}_2)=\lambda$. Thus

$$\left(\Delta\mathbf{s}_1\Delta\mathbf{s}_1^H+\Delta\mathbf{s}_2\Delta\mathbf{s}_2^H\right)(\Delta\mathbf{s}_1+\Delta\mathbf{s}_2)=(\Delta\mathbf{s}_1+\Delta\mathbf{s}_2)\,\lambda. \tag{23}$$

I. Burst Block Orthogonal Coded CPM with Outer Serially Concatenated Code

In a general case of $K > N_s$ with large number of nonzero path difference components, which typically arises as a consequence of the outer serially concatenated code presence, we have to resort to the general solution (17). Also, the individual paths with the minimum determinant does not have to be rotationally invariant. As a consequence the optimal projector will depend on the particular path and we have to treat them as individual cases.

V. Numerical Results

In this section, we provide simulation results for two cases of modulation state termination at the end of the data burst, particularly the unterminated one and the terminated one with 2 nonzero path differences. The block length was intentionally chosen very short, $K=8$. All simulations assume 2×1 MIMO Rayleigh channel with additive Gaussian noise.

First, we analyzed a projection impact on the determinant and the rank of the matrix $\mathbf{R_u}$. For *unterminated* event, we can observe (Fig. 3) that the rank and the minimal determinant of

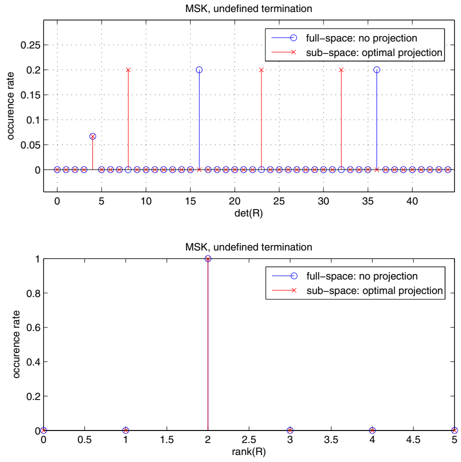

Figure 3. Code rank and determinant spectrum for unterminated trellis.

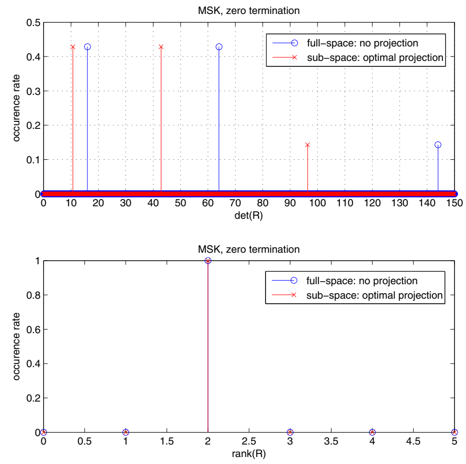

Figure 4. Code rank and determinant spectrum for fix zero state termination

the projected signal is preserved. In this case, the minimal determinant can be evaluated as

$$\begin{aligned}
\min\det\mathbf{R}_p &= \left(\|\Delta\mathbf{s}_1\|^2\right)^2=\left(\left\|(a_1-j,\quad ja_2)^T\right\|^2\right)^2\\
&=\left(1+a_1^2+a_2^2\right)^2=4.
\end{aligned} \tag{24}$$

We can see that the projection did not change the minimum determinant. The determinant and rank occurrence spectrum for the case of *terminated* trellis with 2 nonzero differences is shown on Fig. 4. The rank is, as expected, preserved in both cases.

Fig. 5 and 6 show the BER results. There is always shown the optimal projector case and several suboptimal ones for comparison including the full-space (without any projection) case. We can observe that the choice of wrong projector can cause a substantial loss of the performance. The $\sim$ 1dB shift between the full-space and the optimally projected signals is caused by the change of the overall determinant spectrum

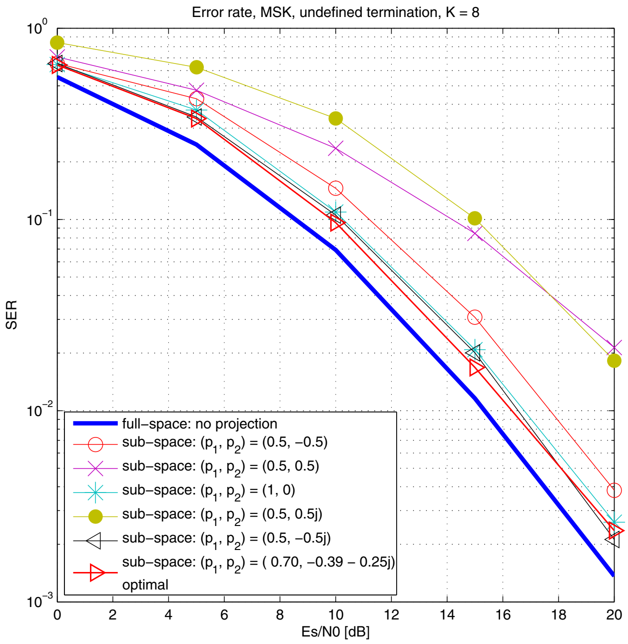

Figure 5. BER for unterminated trellis.

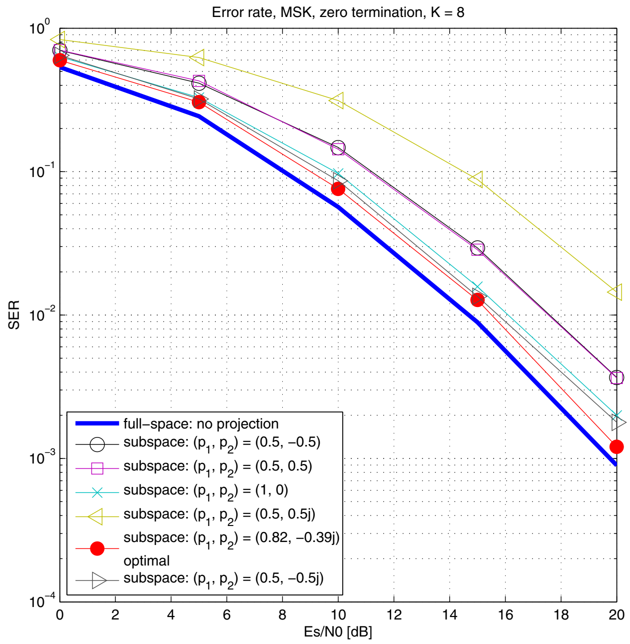

Figure 6. BER for terminated trellis with 2 nonzero differences.

caused by the projection. The projector guarantees only the optimization of the determinant behavior with respect to the most vulnerable path. Other path differences can be affected.

VI. Conclusions

We have systematically derived a closed form solution for the linear *dimensionality reducing* receiver front-end projector applicable for an arbitrary orthogonal design block burst coded CPM modulation. We have exemplified the application on the most simple example of the Alamouti burst coded MSK in 2×1 MIMO channel. The projector preserves the rank and maximizes the minimum determinant of the equivalent projected code. The numerical results show that a proper projection basis optimization is critical for the BER performance. The projector substantially simplifies the receiver processing load by reducing the dimensionality of the problem. A small penalty w.r.t. the full-space processing is caused by change of the determinant code spectrum on higher spectral components. There still remains an open problem of the projector design for the system with outer serially concatenated code which has long and potentially non-rotationally invariant difference sequences in the trellis.

References

[1] B. Vucetic and J. Yuan, *Space-Time Coding*. John Wiley & Sons, 2003.

[2] C.-C. Cheng and C.-C. Lu, “Space-time code design for CPFSK modulation over frequency-nonselective fading channels,” *IEEE Trans. Commun.*, vol. 53, pp. 1477–1489, Sept. 2005.

[3] R. L. Maw and D. P. Taylor, “Space-time coded system using continuous phase modulation,” *IEEE Trans. Commun.*, vol. 55, pp. 2047–2051, Nov. 2007.

[4] R. I. Seshadri and M. C. Valenti, “Capacity-based parameter optimization of bit-interleaved coded CPM with differential detection,” *IEEE Trans. Veh. Technol.*, vol. 57, pp. 345–353, Jan. 2008.

[5] X. Zhang and M. P. Fitz, “Space-time code design with continuous phase modulation,” *IEEE J. Sel. Areas Commun.*, vol. 21, pp. 783–792, June 2003.

[6] A. M. Silvester, R. Schober, and L. Lampe, “Burst-based orthogonal ST block coding for CPM,” in *Proc. IEEE Global Telecommun. Conf. (GlobeCom)*, 2005.

[7] J. Sykora, “Constant envelope space-time modulation trellis code design for Rayleigh flat fading channel,” in *Proc. IEEE Global Telecommunications Conf. (GlobeCom)*, (San Antonio, TX, USA), pp. 1113–1117, Nov. 2001.

[8] J. Sykora, “Symmetric capacity of nonlinearly modulated finite alphabet signals in MIMO random channel with waveform and memory constraints,” in *Proc. IEEE Global Telecommunications Conf. (GlobeCom)*, (Dallas, USA), pp. 1–6, Dec. 2004.

[9] P. A. Laurent, “Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (AMP),” *IEEE Trans. Commun.*, vol. COM-34, pp. 150–160, Feb. 1986.

[10] J. Sykora, “Receiver constellation waveform subspace preprocessing for burst Alamouti block STC CPM modulation,” in *Proc. IEEE Wireless Commun. Network. Conf. (WCNC)*, (Hong Kong), pp. 1–5, Mar. 2007.

[11] J. Sykora, “Linear subspace projection and information waveform manifold based preprocessing for nonlinear multichannel modulation in MIMO channel,” in *Proc. Int. Conf. on Telecommunications (ICT)*, (Cape Town, South Africa), pp. 1–6, May 2005. Invited paper.

[12] J. Sykora, “Information waveform manifold based preprocessing for nonlinear multichannel modulation in MIMO channel,” in *Proc. IEEE Global Telecommunications Conf. (GlobeCom)*, (St. Louis, USA), pp. 1–6, Nov. 2005.

[13] J. Sykora and R. Schober, “Multiplexing precoding scheme for STC-CPM with parametric phase discriminator IWM receiver,” in *Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC)*, (Cannes, France), pp. 1–6, Sept. 2008.

[14] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice-Hall, 1993.

[15] S. Haykin, *Adaptive Filter Theory*. Prentice-Hall, third ed., 1996.