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RESEARCH ARTICLE

Linear diversity precoding design criterion for finite block-fading parallel MIMO channels

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ABSTRACT

A novel optimal two stage coding for finite set of parallel flat-fading MIMO channels with single common information source with specific constant rate requirement is derived. The optimality of suggested coding is achieved in terms of the capacity *versus* outage performance. The well-known optimal coding rule relies on Gaussian codewords spanned over the whole available finite set of parallel channels. We prove that the equivalent preprocessing to the ideal interleaving is to re-code independent parallel channels codewords by a linear inner precoder from a special class of unitary precoders complying with the optimality criterion derived in the paper. Performing such linear mixture of codewords sharing common Gaussian block-wise codebook, the same capacity *versus* the outage is guaranteed without any interleaving over parallel channels. We utilize a virtual multiple access (VMA) channel approach to derive the optimality criterion. Selected precoders with various space-time or time-only domain span were tested against this criterion and we provide the optimality results on variety of the channel parameters. We showed that the temporal processing is the most important one to achieve the optimality of the precoder. A full space-time precoding does not perform better than one which is temporal-only. Copyright © 2010 John Wiley & Sons, Ltd.

KEYWORDS
precoding; MIMO; diversity; channel outage capacity

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1. INTRODUCTION

1.1. Background

A communication system with a finite number of parallel flat-fading multiple-input multiple-output (MIMO) channels is used as the underlying application model for this paper. It might particularly arrive as the delay limited scenario where M time or frequency separated blocks are observed. We assume no channel state information at the transmitter (cf. [1] for an adaptive technique with the channel state information available at the transmitter). Since there is a finite number of channel realizations (understood as parallel channels with shared transmitter, receiver, and information source), the channel observation must be treated as the non-ergodic one, i.e. informationally unstable [2]. We cannot use any more the tools of classical (ergodic) Shannon capacity approach. The performance metric is the capacity *versus* outage. This is the latency limited performance metric. The instantaneous capacity [2] depends on given realizations and it is indeed the random value itself.

A very good explanation of the optimal coding approach to the channel accommodating finite number of realizations is given in Reference [2]. In Reference [3], a very useful and general analogy with compound channel is used in the derivation of a delay limited outage capacity, average and ergodic (Shannon) capacity. The optimal coding relies on long codewords spread over all parallel channel realizations. Such a coding can be understood as an ideal interleaving of symbols from Gaussian codebooks.

Channel precoding is a widely covered topic in recent research. It is investigated in a variety of flavours in terms of the system assumptions and optimization goals. Some authors assume an availability of the channel state information on the transmitter side (unlike this paper). An overview of the precoding strategies under various channel state information availability (perfect, statistical, limited statistical) is treated in Reference [4]. Very recent generalized results based on Tomlinson-Harashima precoding in a multi-user setup are presented in Reference [5]. A number of specific problems arises in the situation of a multi-user system with a limited/none precoding cooperation [6]. Another common widely addressed assumption refers to the optimization goal. Authors frequently assume the ergodic (average) utility function (unlike this paper). This average utility function can have various forms, e.g. ergodic capacity, ergodic diversity, bit error rate, and can be exploited under various symbol alphabet and receiver structure constraints. See References [7–9], and some very recent results in Reference [10].

A particular case of *no channel state information* at the transmitter combined with a *non-ergodic* delay limited optimization goal appears to be neglected by authors. We use the term diversity precoding with the interpretation slightly different from the traditional understanding (e.g. Reference [11]). Our term diversity refers to the system where the information bits are intentionally spread over parallel channels in order to decrease an impact of their individual fading states to the overall performance measured in terms of the *capacity outage*. This is exactly what is done by the linear precoder investigated in this paper.

1.2. Motivation and goals of the paper

Let us assume M parallel independent communication channels, each being equipped with multiple receive and transmit antennas—MIMO channel. The finite set of M channels is called a frame. The channels can be separated by an arbitrary physical principle. Typically, it is a separation in time or frequency domain (time or frequency slots). There are numerous application services (e.g. the streaming data applications) where a drop (an outage) of a single data block disqualifies the whole frame. The probability of the frame outage is given by the probability of the worst channel realization not supporting the rate R required by the service

$$\Pr\{\text{frame outage}\} = \Pr\left\{\bigcup_{m=1}^{M} C(m) < R\right\} \tag{1}$$

where $C(m)$ is the instantaneous capacity in mth channel.

Our goal is to design an inner precoding scheme equalizing the channel capacities in order to minimize the frame outage probability. We want to equalize the channel instantaneous capacities, i.e. to decrease their dispersion, as they are visible to the outer code. This should be compared with Reference [2] where the "average" behaviour is the investigated utility function. This will lead to lower frame outage probability. See Figure 1.

We have generally several options to solve the problem. The first one is the one-stage optimal Gaussian code

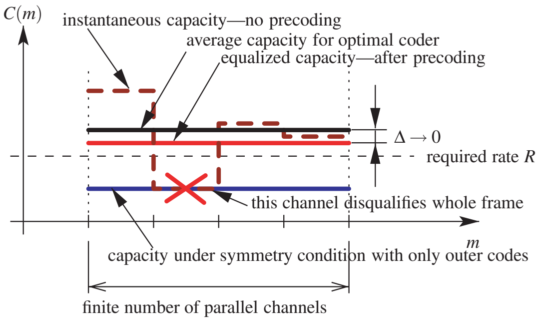

Figure 1. Design goal: instantaneous capacity equalization.

with codewords spanning the whole frame. The code can inherently equalize the influence of the individual channel states by properly spreading the information over the whole frame. However, the complexity of the encoding and decoding would be high due to the codeword spanning the whole frame.

In this paper, we follow the other option of the code design. It uses an outer code with a *common* codebook with the codewords having the length of *one channel* (*block*). The codebook is shared by all channels and the coding/decoding complexity is given only by the single block (channel) length. The outer code is complemented by a low complexity linear inner precoder performing mixing of the channel states inside the frame and thus equalizing the channel capacities visible to the outer code. Our target is an investigation of the design rules for the inner precoder that would equalize the capacities within the frame as much as possible.

The measure of the performance is the frame outage probability. The number of the channels is assumed to be finite and thus the ergodic capacity cannot be used. We compare our two-stage approach with the one-stage one. We use a virtual capacity region [12] as the information theoretic investigation tool to obtain the results.

The paper derives and formulates the optimality *criterion*. The criterion can be used to check whether the given precoder is optimal or not. Unfortunately it cannot give direct hints for the precoder synthesis. However, considering that the system does not use any channel state information at the transmitter, this fact affects only the precoder design and not the run-time operation. Despite of the lack of the synthesis design rules, we provide the results with *qualitative* interpretations that can help to minimize the set of the precoders tested against the criterion. (1) We numerically prove the *existence* of such a precoder (for selected system parameters and arbitrary dimensions of requested precoder). (2) We show that the *temporal* domain precoding is the essential one (compared to the spatial-temporal one) for selected systems. This is caused by the different nature of the spatial and temporal subspaces. The spatial one, unlike the temporal one, contains inherently the "mixing" by the MIMO channel. The received signal

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is a superposition of all signals from transmitted antennas. The temporal subspace requires this feature to be added artificially by the precoder. (3) We show that the precoder itself does not necessarily need to be a complex continuous valued. The discrete valued (Hadamard) precoder also provides optimal results. This has a significant impact on the implementation.

1.3. Outline of the solution

The solution relies on a novel virtual multiple access (VMA) capacity region approach. For the clarity of the further explanation, the detailed outline follows.

- The channel model assumes M independent channels (blocks) in the frame. No form of channel state information is assumed on the transmitter side.
- The channel coder is decomposed into an outer and an inner code.
- The linear precoding (inner code) with a unitary matrix is allowed to mix the output among arbitrary blocks and transmitter antennas in the whole frame.
- Codewords of the outer code are required to be independent over the parallel channels (blocks).
- On the level of outer code output, the system can be seen as a VMA system with the outer codes in the individual channels as separate sources of information.
- The capacity region for such VMA like system is derived.
- The symmetry conditions enforcing a uniform achievable rate for the outer code across the parallel channels are introduced.
- The precoder design criterion guarantees that the pair of precoder plus outer independent code does not perform worse than the direct coder with codeword spanning over the whole frame.

2. SYSTEM MODEL

2.1. One-stage and two-stage channel coding

A one-stage joint encoding (Figure 2) is defined as a mapping $\mathbf{d} \mapsto \mathbf{q}$, where $\mathbf{d}$ is the data vector composed of the sub-vectors for individual channels $\mathbf{d} = [\mathbf{d}_1^{\mathrm{T}}, \ldots, \mathbf{d}_M^{\mathrm{T}}]^{\mathrm{T}}$ and similarly for the codeword vector $\mathbf{q}$. The codeword vectors are drawn from the codebook spanning the whole frame.

The reference case is the one-stage block-wise encoding (Figure 3) where the individual channel encoders do not cooperate at all and all are having the same codebook.

The two-stage encoder (Figure 4) is having independent outer per-block codes sharing the same codebook $\mathbf{c}_m = Q(\mathbf{d}_m)$. Their codewords $\mathbf{c} = [\mathbf{c}_1^{\mathrm{T}}, \ldots, \mathbf{c}_M^{\mathrm{T}}]^{\mathrm{T}}$ are fed into the input of the linear precoder.

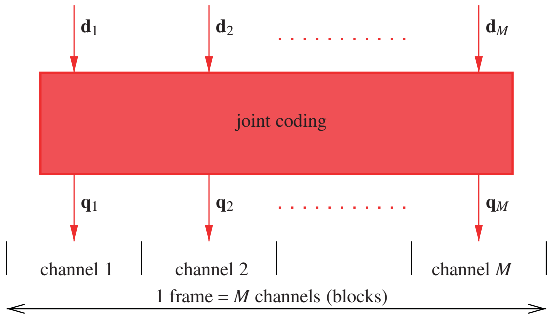

Figure 2. One-stage joint optimal encoder.

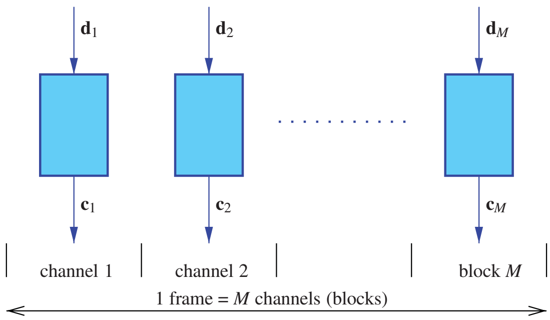

Figure 3. One-stage block-wise encoder.

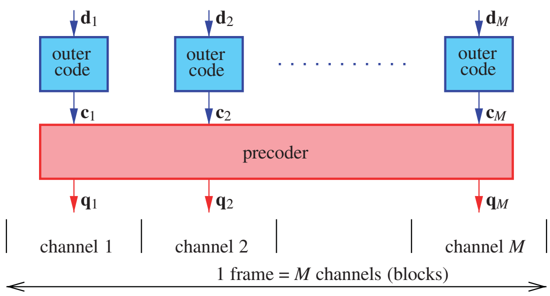

Figure 4. Two-stage encoder with a linear precoder.

2.2. Channel model

All individual channels are MIMO channels with N_T transmit and N_R receive antennas. The input–output model is

$$\mathbf{y}_m = \mathbf{G}_m \mathbf{q}_m + \mathbf{w}_m, \quad m \in \{1, \ldots, M\} \tag{2}$$

where $\mathbf{G}_m$ are independent identically distributed random $N_R \times N_T$ matrices with zero mean complex Gaussian distribution and $\mathbf{w}_m$ is the complex Gaussian additive white noise with σ_w^2 variance per dimension. We define the signal-to-noise ratio as $\Gamma = \mathrm{E}[\|\mathbf{c}_m\|^2]/\sigma_w^2$.

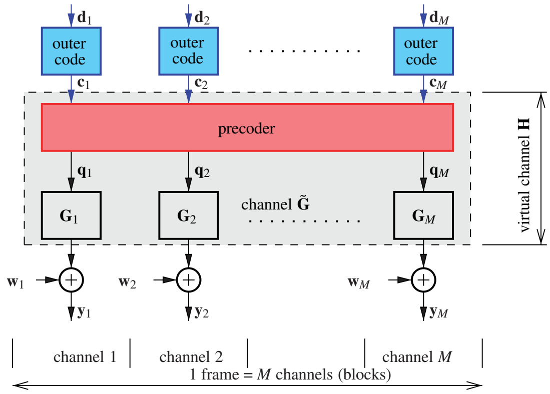

Figure 5. Virtual multiple access channel model.

2.3. Linear precoding and virtual channel

The second stage coding is performed in the signal space domain by a linear operation

$$\mathbf{q} = \mathbf{F}\mathbf{c} \tag{3}$$

The precoding power preserving unitary matrix $\mathbf{F}$ has dimensions $MN_T \times MN_T$. Using the channel input–output relation and the precoder equation, we get

$$\mathbf{y} = \mathbf{G}\mathbf{q} + \mathbf{w} = \mathbf{H}\mathbf{c} + \mathbf{w} \tag{4}$$

where $\mathbf{G}$ is a block-diagonal observed channel matrix

$$\mathbf{G} = \text{diag}[\mathbf{G}_1, \ldots, \mathbf{G}_M] \tag{5}$$

and $\mathbf{w} = [\mathbf{w}_1^{\mathrm{T}}, \ldots, \mathbf{w}_M^{\mathrm{T}}]^{\mathrm{T}}$. The result can be seen as a system with the virtual channel $\mathbf{H}$ from the perspective of the codeword vector $\mathbf{c}$, see Figure 5.

Since this codeword vector is composed from independently encoded sub-vectors $\mathbf{c}_m$ each carrying independent fraction of the data message $\mathbf{d}_m$, we can describe the whole system in a formal form of a *VMA* system

$$\mathbf{y} = \sum_{m=1}^{M} \mathbf{H}_{[m]}\mathbf{c}_m + \mathbf{w} \tag{6}$$

The symbol $\mathbf{H}_{[m]}$ denotes a sub-matrix composed of the columns $(N_T(m-1)+1)$ to $(N_T m)$. The information flow takes a formal form of a multiple access channel with individual symbols $\mathbf{c}_m$ as separate sources of information.

3. VIRTUAL CAPACITY REGION

The VMA model is now used to formulate the information theoretic performance of the various coding models under consideration. Information rates over individual channels R_m must obey the capacity region inequalities

$$\sum_{m\in B} R_m \le I_B(\mathbf{G}, \mathbf{F}); \quad \forall B \subseteq A \tag{7}$$

where $A = \{1, \ldots, M\}$ is the set of all channel indices, B is an arbitrary subset of A and $I_B(\mathbf{G}, \mathbf{F})$ is the mutual information for given subset of virtual users depending on the channel state and precoding matrix. A particular form of this mutual information depends on the coding model chosen.

We require all channels to support a common equal rate R. This will be called a *symmetry condition*. This is the minimum rate required by the service application in each of the channels in the frame.

3.1. First stage outer code only

This coding scheme assumes single stage coding with independent codewords of the individual channels sharing the same codebook. This is a non-optimal case suffering by the capacity fluctuations in full. We use it as a reference.

The sum rate (over all channels in the frame) is given by

$$\sum_{m\in A} R_m \le \sum_{m\in A} I(\mathbf{c}_m; \mathbf{y}) = \sum_{m\in A} I(\mathbf{c}_m; \mathbf{y}_m) \tag{8}$$

The maximum mutual information, the capacity, of the mth channel is [2,13]

$$\begin{aligned} C_m &= \log_2 \det \left(\mathbf{I}_{N_R} + \frac{\Gamma}{N_T}\mathbf{G}_m\mathbf{G}_m^{\mathrm{H}}\right) \\ &= \sum_{j=1}^{N_R} \log_2 \left(1 + \frac{\Gamma}{N_T}\text{eig}_j\left(\mathbf{G}_m\mathbf{G}_m^{\mathrm{H}}\right)\right) \end{aligned} \tag{9}$$

The symbol $\text{eig}_j(\cdot)$ is the jth eigenvalue of the $(\cdot)$ matrix.

The maximum supported rate with the symmetry condition employed is

$$R \le \min_{m\in A} C_m \tag{10}$$

3.2. Optimal ideally interleaved one-stage code

For the case of the optimal joint Gaussian one-stage encoding, the sum rate (over all parallel channels) is constrained by

$$\sum_{m\in A} R_m \le I(\mathbf{q}; \mathbf{y}) \tag{11}$$

Considering our requirement of the symmetric rates, we get

$$R \le \frac{C_{\mathbf{dq}}}{M} \tag{12}$$

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where the total capacity of the frame [2,13] is

$$C_{\mathbf{dq}} = \log_2 \det\left(\mathbf{I}_{MN_R} + \frac{\Gamma}{N_T}\Lambda\right) = \sum_{j=1}^{MN_R} \log_2\left(1 + \frac{\Gamma}{N_T}\mathrm{eig}_j(\Lambda)\right) \quad (13)$$

The Hermitian matrix Λ is given by

$$\Lambda = \mathbf{H}\mathbf{H}^{\mathrm{H}} = \mathbf{G}\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}} \quad (14)$$

3.3. Two-stage coder with common codebook and unitary precoder

The channel appears as the VMA from the perspective of the $\mathbf{c}_m$ codewords. A corresponding capacity region is set by conditions [14]

$$\sum_{m\in B} R_B \le I(\mathbf{c}_B;\mathbf{y}|\mathbf{c}_{A\setminus B}), \quad \forall B \subseteq A \quad (15)$$

where $I(\mathbf{c}_B;\mathbf{y}|\mathbf{c}_{A\setminus B})$ is the conditional mutual information between all $\mathbf{c}_m$ with $m \in B$ conditioned on the knowledge of all $\mathbf{c}_m$ not being part of B. Assuming capacity achieving coding, we get

$$\sum_{m\in B} R_m \le C_B, \quad \forall B \subseteq A \quad (16)$$

where the capacity C_B is the capacity with input channel indices from the set B and the output being the observation $\mathbf{y}$. The drop of channel excitations at indices $m \notin B$ is equivalent to the perfect knowledge of the corresponding channels at the receiver side. The capacity is thus

$$C_B = \log_2 \det\left(\mathbf{I}_{MN_R} + \frac{\Gamma}{N_T}\sum_{m\in B}\Lambda_m\right) = \sum_{j=1}^{|B|N_R} \log_2\left(1 + \frac{\Gamma}{N_T}\mathrm{eig}_j\left(\sum_{m\in B}\Lambda_m\right)\right) \quad (17)$$

where $\Lambda_m = \mathbf{H}_{[m]}\mathbf{H}_{[m]}^H$ is the Hermitian matrix having only the relevant columns corresponding to the active channels.

The maximum achievable rate under the symmetry condition is

$$R \le \min_{B\subseteq A} \frac{1}{|B|} C_B \quad (18)$$

4. OPTIMALITY OF THE LINEAR PRECODER

4.1. Optimality criterion

We want to design a precoder for the two-stage scenario that would perform as well as a one-stage optimal joint encoder. In order to achieve this, the dominant term in Equation (18) must be equal to the capacity of one-stage optimal joint encoding scenario

$$\min_{B\subseteq A} \frac{1}{|B|} C_B = \frac{C_{\mathbf{dq}}}{M} \quad (19)$$

If the precoder complies with this condition, the two stage scheme with block-wise outer encoders (sharing common codebook) performs the same as the frame-long joint optimal one stage coder.

The condition can be graphically interpreted in the virtual capacity region. The straight line $R_1 = \cdots = R_M$ in the code rate hyper-space represents the symmetry condition. The line $R_1 + \cdots + R_M = C_{\mathbf{dq}}$ represents the total rate of one-stage joint coder. The intersection coordinates in each dimension represent minimum symmetric rate of the joint coder (all channels supporting the same minimal rate). If the intersection of those two lines lies inside or on the border of the VMA capacity region then the condition is fulfilled. If it is outside of the VMA capacity region, at least one capacity region condition was dominant and caused the achievable rate to be lower.

Figure 6 shows the capacity region and the sum-rate comparison for the one-stage code and the optimal joint coder. Because of the block structure of the $\mathbf{G}$ matrix, it is clear that $\sum_{m=1}^{M} C_m = C_{\mathbf{dq}}$. Figure 7 shows an optimal and a sub-optimal precoder capacity region examples.

The Mth order left-hand side condition in Equation (19), i.e. $B = A$, is directly equivalent to the capacity of the joint

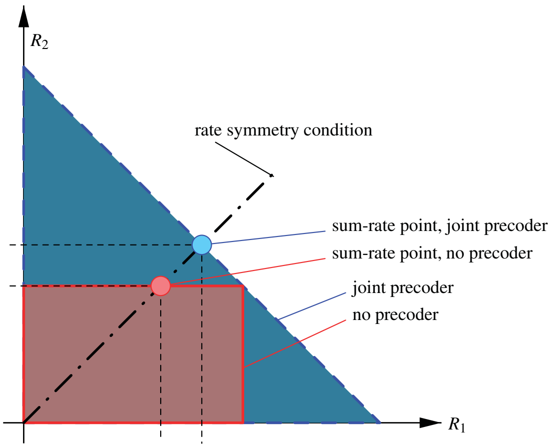

Figure 6. Capacity region: one-stage coder with independent codes and one-stage joint coding.

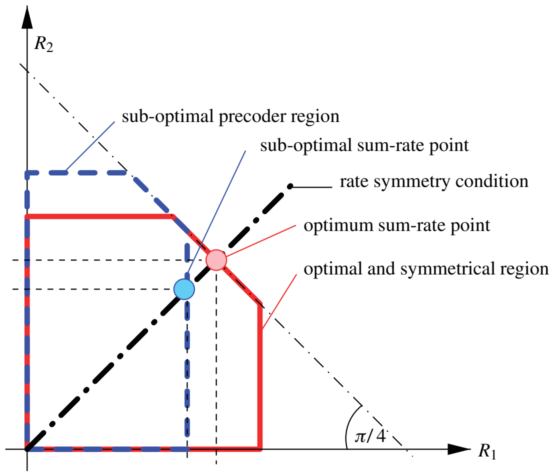

Figure 7. Capacity region: two-stage optimal and sub-optimal precoder.

one-stage coder $C_A = C_{\mathbf{dq}}$. It simply follows from

$$\operatorname{eig}_j\left(\sum_{m\in B}\Lambda_m\right) = \operatorname{eig}_j(\Lambda_B) \tag{20}$$

and corresponding capacity expressions. The matrix $\Lambda_B = \mathbf{H}_B\mathbf{H}_B^H$ is built from the channel matrix $\mathbf{H}_B$ containing only columns from the set B. As a direct consequence of this, the dominant term in Equation (18) cannot have the order of M.

The precoder *optimality criterion* is then

$$\frac{1}{M}C_A \leq \frac{1}{|B|}C_B, \quad \forall B \subset A \tag{21}$$

4.2. Strong and weak optimality

We define the precoder to have *strong* optimality if the precoder fulfils other criteria additionally to satisfying Equation (21). Those criteria are defined as monotonous behaviour of the capacity with respect to the order of the condition

$$\frac{1}{|B_2|}\sum_{j=1}^{|B_2|N_R}\log_2\left(1+\frac{\Gamma}{N_T}\operatorname{eig}_j\left(\sum_{m\in B_2}\Lambda_m\right)\right) \leq \frac{1}{|B_1|}\sum_{j=1}^{|B_1|N_R}\log_2\left(1+\frac{\Gamma}{N_T}\operatorname{eig}_j\left(\sum_{m\in B_1}\Lambda_m\right)\right) \tag{22}$$

In the case of two subsets on indices, the subset with higher cardinality $|B_2| > |B_1|$ always dominates in the criterion.

The precoder is called optimal in a *weak* sense if it just fulfils the criterion (21) regardless of the cardinality of subsets.

5. SELECTED PRECODERS

The optimality criterion (21) does not give direct hints utilizable for a synthesis of the precoder. It can only be used to verify whether the given precoder complies with Equation (21). We have selected number of precoders and evaluated their performance.

5.1. Space-time and time-only precoding

The precoder can distribute (mixture) the information symbols along both spatial and temporal or temporal-only domains. The first option will be called a *full precoder*. It spreads the symbol in both space and time. The precoding matrix $\mathbf{F}$ has non-zero elements generally in all positions. The second option will be called *Kronecker product* (*KP*) *precoder*. It spreads the symbols in time only separately for each particular antenna. The precoding matrix can be constructed from the baseline precoder matrix (describing the temporal spreading) by a Kronecker product expression

$$\mathbf{F} = \mathbf{F}_M \otimes \mathbf{I}_{N_T} \tag{23}$$

5.2. Hadamard precoder

The space-time precoding is represented by an unitary matrix having all non-zero entries. The Hadamard precoder matrix has orthogonal columns with elements ± 1 scaled to meet the power constraint. An example Hadamard matrix of the 4$^{\text{th}}$ order is

$$\mathbf{F} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{24}$$

5.3. Complex field diversity precoder

The second option is so-called *complex field diversity (CFD) precoder*. It is obtained using tools of algebraic number theory, see References [7,15]. It has complex entries with mutually equal magnitudes and orthogonal columns. The CFD precoding is exhaustively treated in Reference [8]. Such design achieves the diversity gain $N_T N_R$ for any finite constellation and therefore it is viewed as the optimal one in the diversity performance. However, in our approach, these precoders are used in a very different channel environment (non-ergodic) and with a strongly different motivation. The CFD precoding matrix of the order N is generally given as well known Fourier orthogonal space. Such precoder consists of elements $\alpha_{ij} = (\mathrm{e}^{\mathrm{j}2\pi(i-1)/M})^{j-1}$ on the ith row and jth column. Again, to preserve the total power it has to be properly rescaled. For our purposes, the most important feature of such complex matrices lies in having entries with equal magnitudes.

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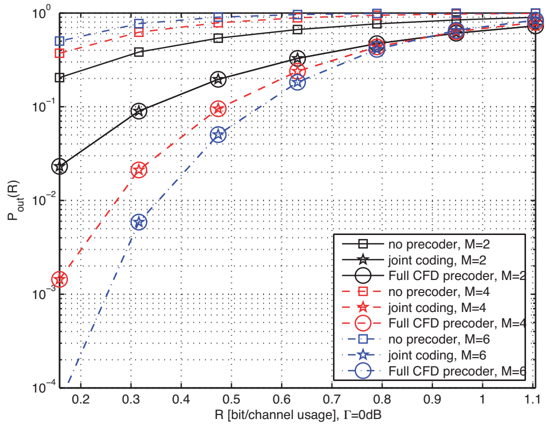

Figure 8. Outage probability, SISO channel, $\Gamma = 0$ dB.

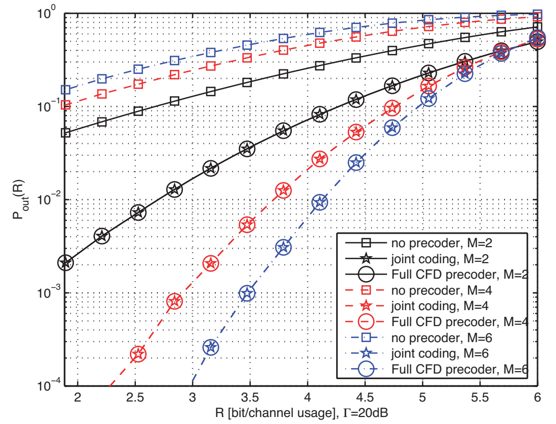

Figure 10. Outage probability, SISO channel, $\Gamma = 20$ dB.

6. NUMERICAL RESULTS

The simulation's numerical results show that there are some special types of precoders that support the criterion (21) but just in a weak sense. We have not found any precoding fulfilling the ultimate criterion in a strong sense. The simulation results show the probability of the outage for a given minimum desired rate per single MIMO parallel channel. It gives us the overall quality of a particular precoder. All simulations were carried out using Rayleigh MIMO fading channel with uncorrelated independent identically distributed blocks in the frame.

Figures 8–10 show the outage probability for the case of SISO channel $N_T = N_R = 1$ and various values of signal-to-noise ratios. There are curves for the scenarios with a no-precoder, a joint one-stage coding and a two-stage coding with a precoder. The CFD precoder performs the same as the joint coding strategy and thus it achieves the *optimality* for all Γ and M.

Figures 11–14 show the situation for MIMO 2×2 channel. We see that the full CFD precoder is *no longer optimal*. The optimality loss decreases with the signal-to-noise ratio. The full Hadamard precoder is performing optimally for some values of M (2,4,8) but particularly for $M = 6$ it is *not* optimal (however the optimality loss is very small). The *Kronecker product* forms of both, CFD and Hadamard, precoders performs *optimally* for all Γ and M.

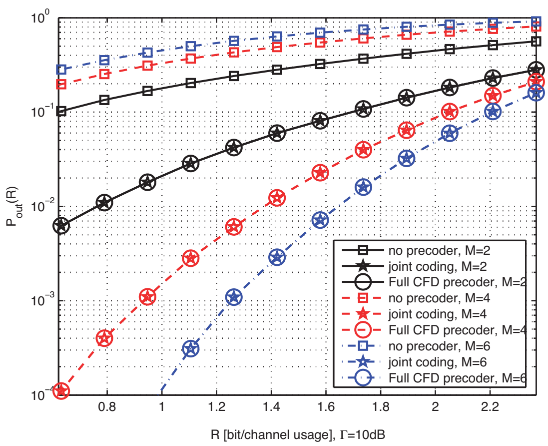

Figure 9. Outage probability, SISO channel, $\Gamma = 10$ dB.

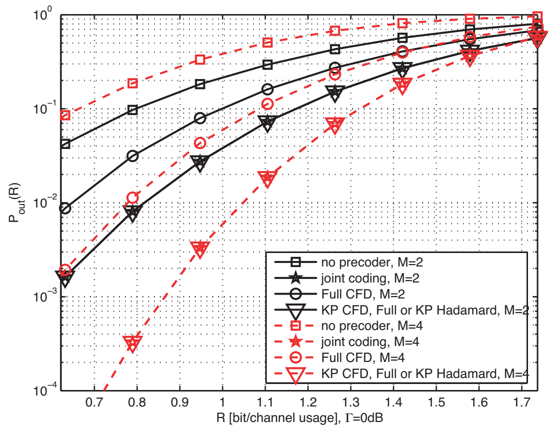

Figure 11. Outage probability, 2×2 MIMO channel, $\Gamma = 0$ dB.

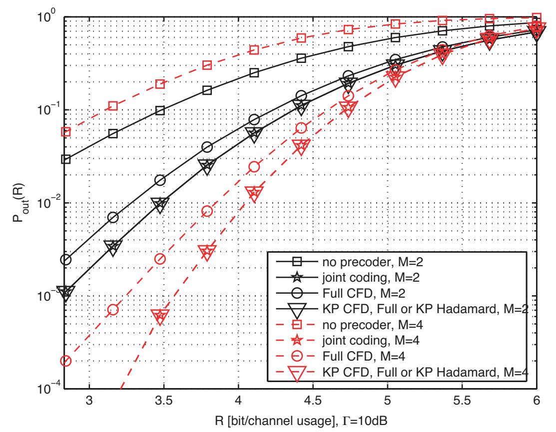

Figure 12. Outage probability, 2×2 MIMO channel, $\Gamma = 10$ dB.

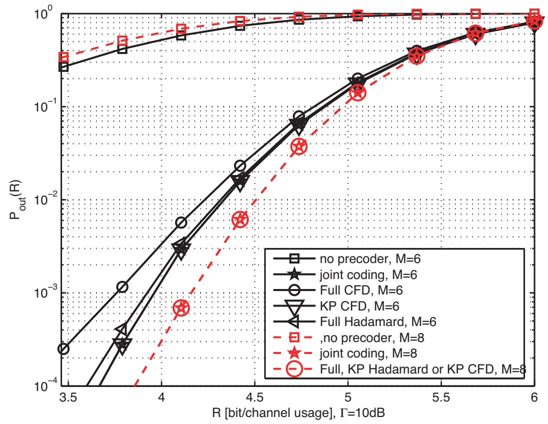

Figure 13. Outage probability, 2 × 2 MIMO channel, Γ = 10 dB.

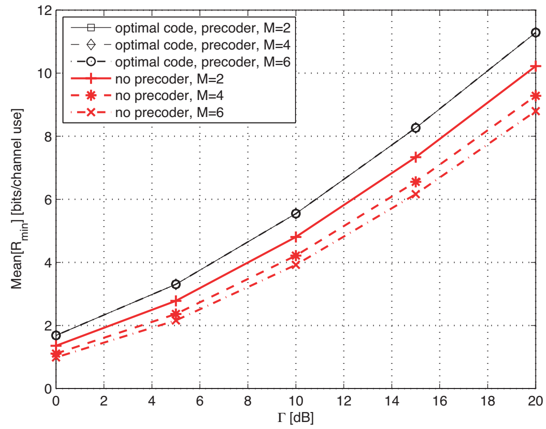

Figure 15. Mean of the maximum rate, 2 × 2 MIMO channel.

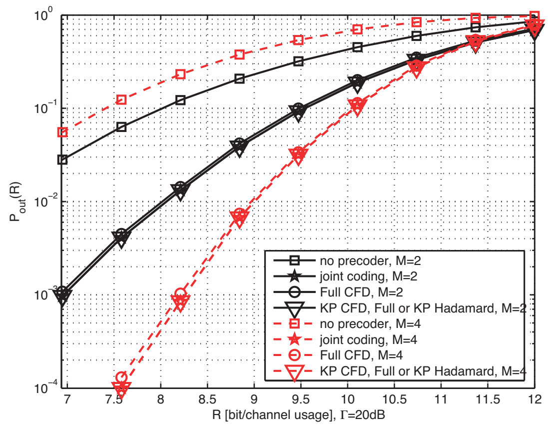

Figure 14. Outage probability, 2 × 2 MIMO channel, Γ = 20 dB.

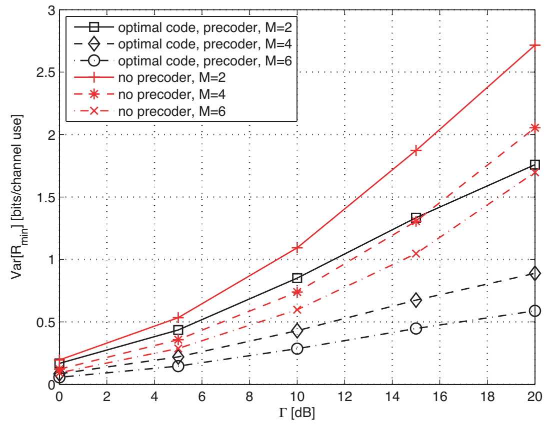

Figure 16. Variance of the maximum rate, 2 × 2 MIMO channel.

Now we turn our attention to the scalar stochastic properties of the maximum common achievable rate. This will nicely demonstrate the equalizing (or stabilizing) effects of the precoder. The mean and the variance of the maximum achievable rate is given in Figures 15 and 16. We can see that all curves of the mean values for the joint (or optimal) coding are identical. This perfectly corresponds to the fact that the ergodic value of the maximum rate does not depend on M. Contrary to that, the higher M corresponds to the lower ergodic maximum achievable rate for no precoding. That is originated by the obvious fact that the minimum for the case with an increasing number of fading occurrences is getting lower in an average. The variance in Figure 16, in a coherence with our expectations, decreases with the number of the parallel channels. The variance for the precoded case is always substantially lower than for the null precoder case.

We now briefly comment on the complexity issues. The system setup with the assumption of no channel state information at the transmitter implies that the precoder is fixed for all transmissions and therefore there is no complexity issue with its run-time evaluation as it is in the case of adaptive transmissions. The precoder is purely given by the channel statistics known *a priori* by finding the matrix $\mathbf{F}$ fulfilling the criterion for a given channel. The evaluation of the precoding matrix multiplication has the complexity given by the number of transmit antennas N_T and the length of the frame M. The full precoder involves multiplication by $MN_T \times MN_T$ matrix. In the case of the Kronecker Product precoder the complexity is substantially reduced to N_T separate multiplications by $M \times M$ matrix. The second case is the most relevant one since we saw that the time domain only (KP) capacity stabilization provides optimal results for all Γ and M.

Assume that the optimal joint coder has codeword length L per-block (L must be large in order for the code to approach the capacity). The overall code will be the space-time $N_T \times (ML)$ code. For the KP precoded case, the coder will be space-time code $N_T \times L$ with each antenna output being multiplied by $M \times M$ matrix. The complexity increase is thus $N_T M$ fold. The multiplications are signal-space operations. However, in the case of the Hadamard KP precoder these can be easily equivalently represented in the discrete code symbols domain, e.g. by a direct modification

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of the encoder. The numerical results show that even very small values of M provide a significant gain in comparison with the no-precoder case. The complexity increase does not appear to be a significant issue. We also need to keep in the mind that the joint precoder complexity increase, when extending the codeword length form L to ML, is likely be higher than the linear one.

7. CONCLUSIONS

We developed two-stage coding strategy with linear unitary inner precoding for the finite set of parallel fading channels. The outer code provides independent codewords and the outer coders share a common codebook. We showed that this scheme can perform the same, in terms of the outage probability, as the one-stage joint coding with codewords spanning the whole frame. The advantage of our scheme is its simpler implementation. The outer codes are having complexity given only by the block length and the inner code is simple linear one. On the opposite, the joint coder would have complexity given by the whole frame length.

The optimality criterion for the precoder is developed based on the VMA region approach. The information-theoretic equivalence between the well-known optimal capacity achieving codebook with Gaussian codewords spanning the whole frame and our approach with linear precoder was developed. The linear combination of the finite number of the channels with outer per-block code is shown to be equivalent to the frame-long code under the fulfillment of the precoder optimality criterion.

We show that the temporal-only CFD and Hadamard precoders (the Kronecker product precoder) are optimal for MIMO case regardless of the number of the parallel channels or signal-to-noise ratio. The spatial-temporal precoding (full precoder) does not generally achieve the optimality. Temporal spreading is more important in achieving optimal outage performance. A full space-time precoding does not perform better than the temporal-only one and thus we can save on the precoder complexity in this aspect.

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