Multiplexing Precoding Scheme for STC-CPM with Parametric Phase Discriminator IWM Receiver

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***Abstract*—A constant envelope Continuous Phase Modulation (CPM) class of Multi-Channel (MC) modulations for multi-antenna transmitters is considered. Each receive antenna receives a superposition of CPM waveforms. The useful signal at the receiver spans a curved space — the Information Waveform Manifold (IWM). Particularly it has the form of a cylinder on a cylinder for 2-component CPM signals. At the receiver, we use a nonlinear preprocessor consisting of a nonlinear projector on the IWM, optional processing on the curved space, and a decoding isomorphism. A particular form of the preprocessor, the 2-component IWM phase discriminator, is analyzed. We show that the discriminator can separate individual instantaneous phases of the component CPM modulations and therefore has multiplexing capabilities with respect to the individual transmitted components. Two parallel channels are created without the need for a time domain processing since the procedure fully relies on the curved space properties of the IWM, and thus can be combined with arbitrary outer space-time coding of MC-CPM. An information theoretic analysis of the multiplexing properties based on the mutual information regions is presented. The discriminator solution has a two-fold ambiguity. We show that the discriminator provides good separation of component phases conditioned on a perfect ambiguity resolution. We propose a transmit spatial-differential precoding scheme that allows the ambiguity resolution.**

I. Introduction

A. Motivation

The theory of *linear* space-time coded modulation for Multiple-Input Multiple-Output (MIMO) channels and corresponding signal processing is widely covered in the literature, e.g. [1], [2]. Space-Time Codes (STCs) applied to nonlinear modulations started to attract the attention of researchers only recently and the corresponding theory covers the area only sparsely. Some performance aspects of Multi-Channel Continuous Phase Modulation (MC-CPM), like information capacity, code design for selected channels, and decoder performance were investigated in e.g. [3], [4], [5], [6], [7], [8].

MC-CPM modulations have a range of very useful properties and provide a very attractive alternative to linear modulations. They are inherently resistant to nonlinear distortion (caused by e.g. C-class amplifier) at the transmitter. As a consequence of the modulation nonlinearity, the dimensionality of the waveform per transmit antenna is higher than one. This opens more dimensions of the channel and also allows to decouple a particular waveform (pulse) shape from its second order properties. Most of those properties are particular to the transmitter side. On the other hand, the receiver side is believed to require more complicated processing (compared to linear modulations) due to the higher dimensionality of the waveform. A completely new approach for receiver processing using specific waveform properties of MC-CPM was proposed in [9], [10]. It builds on fact that the MC-CPM useful signal spans a curved space (the manifold) when passed through a MIMO channel. Particularly, it has the form of a cylinder on a cylinder for 2-component CPM signals. This curved space is called Information Waveform Manifold (IWM). At the receiver, we use a nonlinear preprocessor consisting of a nonlinear projector on the IWM, optional processing on the curved space, and a decoding isomorphism.

The advantage of this approach is twofold. First, it reduces the dimensionality of the processing compared to traditional Euclidean signal space expansion. Second, the useful signal lies on a known curved space. This opens a wide range of nonlinear processing possibilities which are not available in the traditional Euclidean signal space.

B. Goals of this Paper and Contributions

We present a novel multiplexing IWM based receiver signal preprocessing which separates signals from two transmit antennas based purely on the specific properties of MC-CPM IWM *without any time domain* processing. The proposed scheme can be concatenated with an arbitrary outer STC without any mutual interaction. The preprocessing provides two virtual parallel channels and is done separately on each receive antenna. The outer STC operates over that *virtual* MIMO channel which (already at the level of IWM preprocessing) separates signals received from two transmitters in the sample domain without any need for time-domain processing. The STC can utilize the virtual channels for ST multiplexing or for fully exploiting the channel diversity.

The closest scenario in the literature to the considered one is the layered multiplexing with multiple CPM in [11]. However, the scheme in [11] uses time domain receiver processing to separate the signals. This is achieved either by the orthogonality of such signals or by successive decoding frequently in combination with other techniques (e.g. equalizing and/or zero forcing). The IWM preprocessor does not need that. It leaves

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the time domain dimension to be used by the outer code for its own optimization goal and does not have to be optimized for the multiplexing itself, c.f. Sec. III.

The first goal of the paper is the information-theoretic assessment of the multiplexing properties of the 2-component IWM phase discriminator based on mutual information regions. We derive these regions using a linearized estimation error model and based on this we define the multiplexing separation ratio. It shows very good separation of the information in both virtual channels. We also compare the approximate densities used in the evaluation with the exact solution, see Sec. V.

The second goal of the paper is the design of a practical precoding scheme that resolves the ambiguity of the IWM phase discriminator and that could be used to demonstrate the basic functionality of the idea, see Sec. IV.

II. System Model and Definitions

A. Multi-Channel CPM Modulation

Multi-Channel CPM modulation is a practically important special case of Nonlinear Multichannel Modulation possessing the important property of constant envelope. The i-th transmitter produces the complex envelope signal $s_i(t) = \exp(j\phi(t, \mathbf{q}_i))$ where $\mathbf{q}_i$ is a vector of real valued discrete finite alphabet channel symbols $q_{i,n}$ and $\phi(t, \mathbf{q}_i) = \sum_n q_{i,n}\beta(t-nT_S)$ is the instantaneous phase of the signal. The phase function $\beta(\centerdot)$ is continuous and controls the trajectory of the phase. It determines the particular type of CPM. Its first derivative is called the frequency pulse $\mu(t) = \partial\beta(t)/\partial t$. Popular choices for the frequency pulse are rectangular pulses (CPFSK), raised cosine pulses (RC-CPM), or the convolution of a rectangular pulse and a Gaussian pulse (GMSK). We assume for simplicity, that the modulation index (a multiplicative scaling constant of the instantaneous phase) is inherently contained in a proper scaling of the channel symbols and phase function.

B. MIMO Channel

The transmitted signal passes through an (N_T, N_R) MIMO flat fading channel. The signal *preprocessing* is done separately at each receive antenna. The separate preprocessing applies only for nonlinear IWM preprocessing. The actual *space-time outer decoding* can operate jointly on all receive antennas. For the sake of notational clarity, the receive antenna *index* will be *dropped* in the rest of the paper. The signal at an arbitrary receive antenna consist of useful signal $a(t)$ which is a weighted superposition of signals from all transmit antennas and additive noise $x(t) = a(t) + w(t)$. The noise $w(t)$ is the complex rotationally invariant white Gaussian noise passed through the receiver front-end selectivity (not affecting the useful signal) thus having finite variance σ_w^2 and zero mean. The useful signal is $a = Ae^{j\alpha} = \sum_{i=1}^{N_T} a_i$ where $h_i s_i(t, \mathbf{q}_i) = a_i = A_i e^{j\alpha_i}$, $h_i = H_i e^{j\eta_i}$ are channel coefficients, and $\alpha_i = \phi_i + \eta_i$ are the composite instantaneous phases of the modulator and the channel, $A_i = H_i$. For the rest of the treatment, the channel coefficients h_i are assumed to be *known*. Assuming known h_i, we can equivalently consider the equivalent system with composite (transmitted signal with channel) amplitudes A_i and phases α_i (see Fig. 3). All quantities are generally depending on time but we do not denote that explicitly. In the remaining part of the paper, we consider a system with two transmit antennas. The resulting equivalent model for the signal at an arbitrary receive antenna is $x = Xe^{j\psi} = Ae^{j\alpha} + w = A_1 e^{j\alpha_1} + A_2 e^{j\alpha_2} + w$.

III. 2-Component IWM Phase Discriminator

A. Information Waveform Manifold

The useful part $a(t)$ of the received signal forms a curved (nonlinear) topological space, the manifold. Since it is formed by the information carrying signal we call it Information Waveform Manifold (IWM). A particular form of the IWM for the *constant envelope* class of modulation allows a novel approach to signal processing compared to the traditional Euclidean signal space decomposition based ones. See [9], [10] for the background information on IWM preprocessing. Here, we summarize the main points and extend the results of [9] by an ambiguity-free region of operation.

A particular form of the 2-component ($N_T = 2$) CPM signal IWM is a cylinder-on-cylinder (see [9]). Every manifold can be parametrized by its parametric space. Signal processing on the IWM has two main advantages. First, the dimensionality of the parametric space of the IWM for 2-component CPM is equal to 3 (including time domain). The most obvious parametrization is $\{\alpha_1, \alpha_2, t\}$. This dimensionality does not depend on the particular CPM modulation type and it is typically much lower than the dimensionality of the traditional Euclidean signal space expansion. Second, the useful signal lies on a known curved space which opens a wide range of nonlinear processing possibilities that are not available in the traditional Euclidean signal space. The concept of using the IWM receiver preprocessing consists of three basic steps: (1) Nonlinear projection on IWM, (2) Signal processing on the IWM (optional), and (3) Isomorphism between IWM and decoder metric.

1) Nonlinear projector on IWM : The nonlinear projection operator $z(t) = T[x(t)]$ is the actual operation that reduces the dimensionality of the problem. It can be performed as an operator, i.e. the mapping including the time domain, or as function $z(t) = T(x(t))$. The latter, called a *sampled* space projector, is generally suboptimal but much simpler. The full space projector can be replaced by the sampled one followed by a proper signal space processing on the IWM.

A particular form of the sample space IWM projector for two component CPM signals can be based on a *constrained Maximum Likelihood (ML)* criterion [9] $z = \arg\max_{\check{a}\in\Psi} p(x|\check{a})$ where $\Psi = \{A_1 e^{j\alpha_1} + A_2 e^{j\alpha_2}\}_{\alpha_1,\alpha_2}$ is the useful signal IWM. The solution has the form of a parametric limiter $z = \chi(X)e^{j\psi}$ with AM/AM conversion $Z = \chi(X)$

$$\chi(X) = \begin{cases} A_1 + A_2, & X > A_1 + A_2 \\ |A_1 - A_2|, & X < |A_1 - A_2| \\ X, & \text{elsewhere} \end{cases} . \quad (1)$$

Amplitudes A_1, A_2 are assumed to be known. The projector generally does not form a sufficient statistics. However, it was shown in [9] that the information loss is almost negligible and appears only at very low signal-to-noise ratios (SNRs).

2) Signal processing on the IWM : Signal processing on the IWM is an optional step. This allows the incorporation of *time* domain processing (not performed otherwise) into the preprocessor. It can replace the true full (value and time domain) IWM projection operator properly utilizing the properties of the time-domain behavior of the signal on the IWM. The signal processing on the IWM must use tools from differential geometry, c.f. [12], [13], [14], [15], to perform the signal processing operations (intrinsic, i.e. those using differential geometry tools vs. extrinsic operations, i.e. those performed in the embedding Euclidean space with subsequent projection on the manifold). The IWM is a curved space. The curvature generally stems from two sources. The first is a nonlinearity of phase function $\beta(t)$ in time. The second one appears specifically in MIMO system where superposition of two cylindrical IWMs of CPM curves the space (see [9]).

3) Isomorphism between IWM and decoder metric: After the IWM processing is completed, we must provide a suitable metric for the outer space-time decoder. This isomorphism can have various forms depending on the particular needs of the space-time decoder. A decoding metric composed of individual component phase estimates $\hat{\alpha}_1, \hat{\alpha}_2$ is clearly one of the attractive choices. The isomorphism can be constructed using a geometric approach [9] (solving the triangular problem).

B. 2-Component IWM Phase Discriminator

A particular form of IWM receiver preprocessing is the 2-Component IWM Phase Discriminator. It consists of a nonlinear sample space projector on the IWM implemented as a parametric limiter (1) and a decoder metric formed as estimates of composite phases α_1, α_2 for the actual *projected* point of the *received* signal $z = Ze^{j\psi}$. The solution has two ambiguity domains $[\hat{\alpha}_1 - \psi, \hat{\alpha}_2 - \psi] \in \{\pm[\check{\alpha}_1, \check{\alpha}_2]\}$, $\check{\alpha}_1 \in [0, \pi]$, $\check{\alpha}_2 \in [-\pi, 0]$

$$\check{\alpha}_1 = -\arccos\left(\left(A_1^2 - A_2^2 + Z^2\right) / (2A_1 Z)\right), \quad (2)$$

$$\check{\alpha}_2 = \arccos\left(\left(A_2^2 - A_1^2 + Z^2\right) / (2A_2 Z)\right). \quad (3)$$

C. Ambiguity-Free Region and Unwrapped Solutions

One of the problems of the solutions in (2), (3) is the two-fold ambiguity. Fig. 1 (a) shows that an ambiguity free solution (solution 1 chosen) of the triangular problem is defined by a condition for the true composite phases $0 \le \Delta\alpha < \pi$ where $\Delta\alpha = (\alpha_2 - \alpha_2)_{\bmod 2\pi}$ and solution 2 is under condition $\pi \le \Delta\alpha < 2\pi$. This is shown on Fig. 1 (b). The solution for the real received signal is then

$$\hat{\alpha}_1 = \psi - \arccos\left(\left(A_1^2 - A_2^2 + Z^2\right) / (2A_1 Z)\right), \quad (4)$$

$$\hat{\alpha}_2 = \psi + \arccos\left(\left(A_2^2 - A_1^2 + Z^2\right) / (2A_2 Z)\right). \quad (5)$$

The ambiguity free region at the level of composite phases α_i can be equivalently achieved by a correct ambiguity resolution using proper precoding at the transmitter side (restricting the values of ϕ_i).

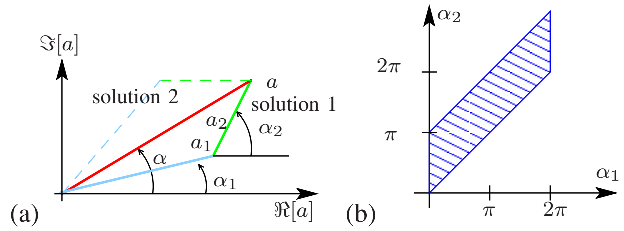

Figure 1. (a) Ambiguous solution of the triangular problem. (b) Ambiguity-free region for solution 1.

The solutions (4), (5) can be positive or negative. It is useful to define unwrapped solutions shifted $\bmod 2\pi$ equivalently such that the result is in the $[0, 2\pi)$ range. We define the mth ($m \in \{1, 2\}$) solution for the ith phase as $\hat{\alpha}_i^m$ and similarly the unwrapped versions. They are ($\mathcal{U}$ is the Unit Step function) $\hat{\alpha}_{iu}^m = \hat{\alpha}_i^m + 2\pi\mathcal{U}(-\hat{\alpha}_i^m)$.

IV. SPATIAL DIFFERENTIAL PRECODER

The ambiguity resolving precoding is based on the following idea. Given the phase shift difference of the channel propagation from both transmitter antennas $\Delta\eta = (\eta_2 - \eta_1)_{\bmod 2\pi}$ and phase differences of the two transmitted signals $\Delta\phi = (\phi_2 - \phi_1)_{\bmod 2\pi}$, $\Delta\alpha = \Delta\phi + \Delta\eta$ holds. Given that we know $\Delta\eta$ (perfect channel state information at the receiver is assumed) and we would limit the range of allowed $\Delta\phi$ (this is in fact the definition of possible waveform phase precoder codewords) then we can decide on the expected range of $\Delta\alpha$ and thus we can decide which solution (1 or 2) to use for the discriminator. One of the possible options to achiever this is to limit the transmitted signal to

$$0 \le \Delta\phi < \pi. \quad (6)$$

a) Unresolvable: If *both* solutions $\Delta\alpha^1, \Delta\alpha^2$ fulfill $0 \le (\Delta\alpha^i - \Delta\eta)_{\bmod 2\pi} < \pi$, $i \in \{1, 2\}$, i.e. for both of them, the transmitted signal condition (6) is feasible, we *cannot* distinguish (resolve) which solution to pick. In the opposite case, we can resolve the solution. The unresolvable case is unresolvable only when taking into account one sample only and appears only for limited range of $\Delta\alpha^i, \Delta\eta$ values. We have several options. The particular value can be *erased* (erasure channel) with a similar effect as for punctured coding. This would *keep* the processing only *in the value domain*. Besides that, we still have the temporal domain at our disposal or any other form of a priori information. The a priori information (codeword structure) on the correct solution from the previous sample in time domain can be used for this in a similar manner as e.g. the PLL tracking loop keeps a lock on one of the stable nodes.

b) Solution 1 : The correct one is solution 1 if $0 \le (\Delta\alpha^1 - \Delta\eta)_{\bmod 2\pi} < \pi$, $\pi \le (\Delta\alpha^2 - \Delta\eta)_{\bmod 2\pi} < 2\pi$.

c) Solution 2 : The correct one is solution 2 if $\pi \le (\Delta\alpha^1 - \Delta\eta)_{\bmod 2\pi} < 2\pi$, $0 \le (\Delta\alpha^2 - \Delta\eta)_{\bmod 2\pi} < \pi$.

A particular way how to fulfill transmit condition (6) is to use a *spatial differential CPM modulator* $\phi_2 = \phi_1 + \Delta\phi$ where ϕ_1 is the full information carrying phase of arbitrary

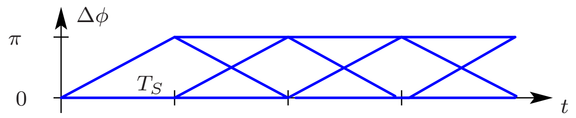

Figure 2. Phase tree of AMI precoded spatial phase difference.

CPM at the first transmit antenna. The difference between the transmitters phases $\Delta\phi$ carries the second information stream. We just have to make sure that this phase is continuous and within limit (6). Then the second antenna signal will also be a CPM signal. The limit (6) can be achieved by using an AMI (Alternate Mark Inversion) precoder for the data stream of symbols $c_{2,n} \in \{0, 1\}$. The output and the state equation of the differential precoder is $q_{\Delta,n} = c_{2,n}\sigma_{2,n}$, $\sigma_{2,n+1} = \sigma_{2,n}(1 - 2c_{2,n})$ where $\sigma_{2,n} \in \{\pm 1\}$ and $q_{\Delta,n} \in \{-1, 0, 1\}$. The phase (see Fig. 2) $\Delta\phi = 2\pi \sum_n q_{\Delta,n}\beta(t - nT_S)$ will satisfy condition (6) if the phase shaping function has one symbol correlation length and $\beta(t) \leq 1/2$. The initial state is $\sigma_{2,0} = 1$.

V. Multiplexing Properties of the Discriminator

This section analyzes the multiplexing properties of the IWM phase discriminator based on an information-theoretic mutual information region approach. For the purpose of the analysis in this section, we assume that the composite phases fulfill the condition $0 \leq \Delta\alpha < \pi$, i.e. we assume solution 1 to be the correct one. This is equivalent to the *perfect ambiguity resolution*, which is a simplifying assumption that makes the evaluation tractable. It frees us from the necessity to use the erasures (or any other technique) to handle the unresolvable case (Sec. IV) at the receiver side of the processing.

A real system will suffer by the presence of the unresolvable case (see Sec. IV) treated by the erasures with the outer code handling those erasures. The probability of the erasures will have a negative influence (depending on a particular CPM type and the outer code) on the information throughput and potentially also on the multiplexing separation. An evaluation of this degradation is subject of the further work.

A. Multiplexing IWM Processing

In the preceding text, we defined the IWM based phase discriminator (separator) of the composite signal phases. The information carrying phases $\phi_i(t, \mathbf{q}_i)$ are separated at the receiver (at the level of composite phases $\alpha_i = \phi_i + \eta_i$) thus allowing information multiplexing of the individual data $\mathbf{q}_i$ streams (see Fig. 3). It is very important to stress that the whole multiplexing processing (under the perfect ambiguity resolution) of the IWM discriminator is purely done in the *value domain without any time-domain processing*. Therefore it can be easily combined with an arbitrary outer STC performed on phases ϕ_i.

B. Mutual Information Region

The *informationally equivalent* system in Fig. 3 is a system with two inputs α_1, α_2 (informationally equivalent inputs obtained from the knowledge of channel h_1, h_2 and ϕ_1, ϕ_2) and

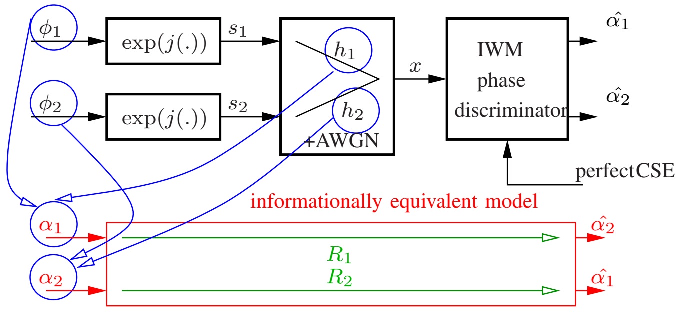

Figure 3. System model of multiplexing IWM processing at one Rx antenna.

two outputs $\hat{\alpha}_1, \hat{\alpha}_2$. Let the R_i be the information rate between $\{\alpha_i, \hat{\alpha}_i\}$ defined separately for $m \in \{1, 2\}$ and C_1, C_2, C_{12} mutual information values defined by the mutual information $I(\cdot;\cdot)$ functions below. The bounding conditions for R_i are given by the mutual information region (the quantities are not the capacities since we do not optimize over the input distribution)

$$R_1 < C_1 = I(\alpha_1; \hat{\alpha}_1 | \alpha_2), \tag{7}$$

$$R_2 < C_2 = I(\alpha_2; \hat{\alpha}_2 | \alpha_1), \tag{8}$$

$$R_1 + R_2 < C_{12} = I(\boldsymbol{\alpha}; \hat{\boldsymbol{\alpha}}), \tag{9}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]^T$ and $\hat{\boldsymbol{\alpha}} = [\hat{\alpha}_1, \hat{\alpha}_2]^T$. This region is somewhat similar to that of the classical multiple access channel (see [16]). It has however one subtle but *important difference*. The conditional mutual information in (7) and (8) has the output variable corresponding only to *one of the outputs*, which is different from the multiple access case where both outputs are considered. The mutual information interpretation is the amount of information shared between a single input and a single output given that the other one is not present or it is known (which is equivalent — any of its influence can be subtracted). This is obviously the most favorable situation and rate R_i cannot exceed this value under any condition. The sum-rate condition evaluates the total information throughput between both inputs jointly and both outputs where encoder and decoder can use joint processing for both channels.

The region allows to quantitatively assess the multiplexing properties of the system, i.e. the level of separation of the virtual channels $\hat{\alpha}_1(\alpha_1)$ and $\hat{\alpha}_2(\alpha_2)$. The situation is illustrated on Fig. 4. Perfect multiplexing allows communication in channel $\hat{\alpha}_1(\alpha_1)$ with rate R_1 without any influence on rate R_2 in the second channel (and vice versa).

We define the *multiplexing separation ratio* as $\kappa = C_{12}/(C_1 + C_2)$. Clearly, any value $\kappa < 1$ causes the region (Fig. 4) to have the upper right corner cut-out because of the C_{12} condition. This corresponds to imperfect channel separation. Values $\kappa > 1$ indicate separation with nonzero "safety" margin. The coefficient κ describes a quality of channel separation, the lower κ the worse separation.

C. Approximation of the Discriminator Output

The evaluation of the quantities C_1, C_2, C_{12} in an exact manner is difficult. The difficulty arises from two sources. The

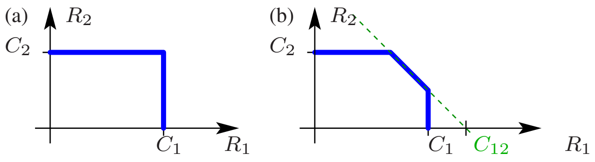

Figure 4. Mutual information region with (a) perfect multiplexing (separation), (b) imperfect multiplexing.

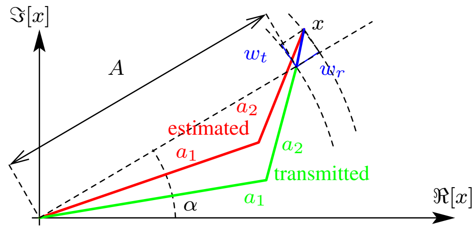

Figure 5. Tangential and radial noise components.

first is nonlinear projector $z = \chi(X)e^{j\psi}$ and the second one comes from the nonlinear functions in (4), (5). Therefore, we first try to find a tractable approximation of the discriminator output.

Both problems can be overcome assuming medium to large SNR. In this case the information loss caused by the nonlinear projector is negligible [9]. We can then approximate the solution by leaving out the projector (i.e. considering x instead of z in (4), (5))

$$\hat{\alpha}_1 \approx \psi - \arccos\left(\left(A_1^2 - A_2^2 + X^2\right)/\left(2A_1X\right)\right), \quad (10)$$

$$\hat{\alpha}_2 \approx \psi + \arccos\left(\left(A_2^2 - A_1^2 + X^2\right)/\left(2A_2X\right)\right). \quad (11)$$

We are in fact not considering those noise values causing the received signal to go outside the $|A_1 - A_2| \le X \le A_1 + A_2$ range. This happens rarely for sufficiently high SNR.

The second problem is solved by utilizing the *rotational invariance* of the noise w. We use its tangential w_t and radial w_r components. They have the same properties as the original real and imaginary parts $w = w_{\Re} + jw_{\Im}$. See Fig. 5. Clearly $X \approx A + w_r$, for $w_t \ll A$. It holds that $\psi = \alpha + \phi$ and $\tan(\varphi) = w_t/(A + w_r)$. For reasonable SNR we also have $w_r \ll A$ and $\tan(\varphi) \approx w_t/A$. The angle ψ is then purely a function of tangential component. The second part of (10), (11) ($\breve{\alpha}_i$) is on the other hand a function of w_r only. Thus,

$$\hat{\alpha}_1 \approx \alpha + \varphi(w_t) - \arccos\left(\frac{A_1^2 - A_2^2 + (A + w_r)^2}{2A_1(A + w_r)}\right) \quad (12)$$

$$\hat{\alpha}_2 \approx \alpha + \varphi(w_t) + \arccos\left(\frac{A_2^2 - A_1^2 + (A + w_r)^2}{2A_2(A + w_r)}\right) \quad (13)$$

Next, the nonlinearities of (12), (13) are approximated by *first order Taylor expansion* w.r.t. w_r, w_t. The coefficients at the linear term are get as the first derivatives w.r.t. w_r, w_t. The error of this linear approximation will be small for sufficiently high SNR. The linear approximation $\hat{\tilde{\alpha}}_1, \hat{\tilde{\alpha}}_2$ is composed of the *noise-free components* α_1, α_2 and the *error* $\Delta\alpha_i(w_r, w_t)$ fully capturing the influence of the noise $\hat{\alpha}_1 \approx \hat{\tilde{\alpha}}_1 = \alpha_1 + \Delta\alpha_1, \hat{\alpha}_2 \approx \hat{\tilde{\alpha}}_2 = \alpha_2 + \Delta\alpha_2$ where the *linearized estimation error* is ($\hat{\tilde{\boldsymbol{\alpha}}} = [\hat{\tilde{\alpha}}_1, \hat{\tilde{\alpha}}_1]^T$, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]^T$)

$$\Delta\alpha_1 = \frac{w_t}{A} + w_r \frac{A^2 - A_1^2 + A_2^2}{2A^2A_1\sqrt{1 - \frac{(A^2 + A_1^2 - A_2^2)^2}{4A^2A_1^2}}}, \quad (14)$$

$$\Delta\alpha_2 = \frac{w_t}{A} - w_r \frac{A^2 + A_1^2 - A_2^2}{2A^2A_2\sqrt{1 - \frac{(A^2 - A_1^2 + A_2^2)^2}{4A^2A_2^2}}}. \quad (15)$$

D. Approximation of the Region

The linearized model can be now used for evaluation of C_1, C_2, C_{12}. The linearized estimation error $\Delta\boldsymbol{\alpha} = [\Delta\alpha_1, \Delta\alpha_2]^T$ has conditional probability density function (PDF) $p_{\Delta\boldsymbol{\alpha}|\boldsymbol{\alpha}}(\Delta\boldsymbol{\alpha})$ which is zero mean Gaussian with covariance matrix

$$\mathbf{C}_{\Delta\boldsymbol{\alpha}} = \frac{\sigma_w^2}{2} \frac{\begin{bmatrix} -4A_2^2 & 2(A^2 - A_1^2 - A_2^2) \\ 2(A^2 - A_1^2 - A_2^2) & -4A_1^2 \end{bmatrix}}{A^4 + (A_1^2 - A_2^2)^2 - 2A^2(A_1^2 + A_2^2)}. \quad (16)$$

The covariance is inherently a function of composite phases $\boldsymbol{\alpha}$ through the variable $A(\boldsymbol{\alpha})$. A marginalization of $p_{\Delta\boldsymbol{\alpha}|\boldsymbol{\alpha}}(\Delta\boldsymbol{\alpha})$ provides $p_{\Delta\alpha_i|\boldsymbol{\alpha}}(\Delta\alpha_i)$.

Capacities obtained from the linearized model are $\tilde{C}_i = I(\hat{\tilde{\alpha}}_i; \alpha_i|\bar{\alpha}_i) = \mathcal{H}[\hat{\tilde{\alpha}}_i|\bar{\alpha}_i] - \mathcal{H}[\hat{\tilde{\alpha}}_i|\boldsymbol{\alpha}]$, $\tilde{C}_{12} = I(\hat{\tilde{\boldsymbol{\alpha}}}; \boldsymbol{\alpha}) = \mathcal{H}[\hat{\tilde{\boldsymbol{\alpha}}}] - \mathcal{H}[\hat{\tilde{\boldsymbol{\alpha}}}|\boldsymbol{\alpha}]$ where $\mathcal{H}[\alpha]$ is the entropy of the random variable α, $i \in \{1, 2\}$, and $\bar{\alpha}_i$ is a complementary input for α_i (e.g. $\bar{\alpha}_1 = \alpha_2$). The multiplexing separation ratio for the linearized model is $\tilde{\kappa} = \tilde{C}_{12}/(\tilde{C}_1 + \tilde{C}_2)$. Capacities are evaluated with the input distribution of $\boldsymbol{\alpha}$ with independent components with *uniform* distribution over the *ambiguity-free region* (Fig. 1) and a *Gaussian* linearized estimation error. Details of the evaluation are not shown here because of the space limitations.

E. Exact Solution for $p(\hat{\boldsymbol{\alpha}}|\boldsymbol{\alpha})$

All entropies needed for the mutual information region evaluation (7), (8), (9) involve density $p(\hat{\boldsymbol{\alpha}}|\boldsymbol{\alpha})$. We evaluate this density in an exact form in order to compare it with its approximate form obtained by the linearization. We show the results in a condensed form omitting details due to the space restrictions. We start with the joint density (see [9]) of the amplitude and phase of the limiter output ($\delta(\cdot)$is Dirac delta function) $p(Z, \psi|\boldsymbol{\alpha}) = p(\psi|\boldsymbol{\alpha}, Z_1)\delta(Z - Z_1) + p(\psi|\boldsymbol{\alpha}, Z_2)\delta(Z - Z_2) + p(X = Z, \psi|\boldsymbol{\alpha})\left(\mathcal{U}(Z - Z_1) - \mathcal{U}(Z - Z_2)\right)$ where $p(\psi|\boldsymbol{\alpha}, Z_1) = \int_0^{Z_1} p(X, \psi|\boldsymbol{\alpha})\, dX$, $p(\psi|\boldsymbol{\alpha}, Z_2) = \int_{Z_2}^{\infty} p(X, \psi|\boldsymbol{\alpha})\, dX$ and $Z_1 = |A_1 - A_2|$, $Z_2 = A_1 + A_2$. The density $p(X, \psi|\boldsymbol{\alpha})$ is the Gaussian one transformed into polar coordinates $p(X, \psi|\boldsymbol{\alpha}) = X p_w(Xe^{j\psi} - a(\boldsymbol{\alpha}))$ where $p_w(w) = 1/(\pi\sigma_w^2)\exp(-|w|^2/\sigma_w^2)$. The *border* partial densities $p(\psi|\boldsymbol{\alpha}, Z_1)$, $p(\psi|\boldsymbol{\alpha}, Z_2)$ can be derived in closed form.

Next, we *restrict* the evaluation of the density only to the density in the *inner region* of the limiter output where $Z \in (Z_1, Z_2)$. This density component is then $p_i(Z, \psi|\boldsymbol{\alpha}) =$

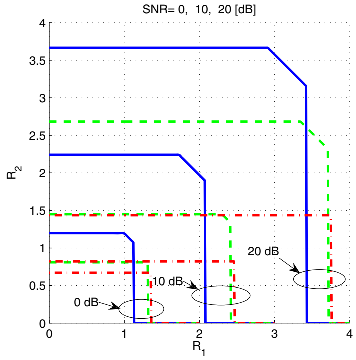

Figure 6. Region of linearized estimation model. (a) $A_1 = 1$ (solid), (b) $A_1 = 3$ (dashed), (c) $A_1 = 10$ (dash-dotted), $A_2 = 1$.

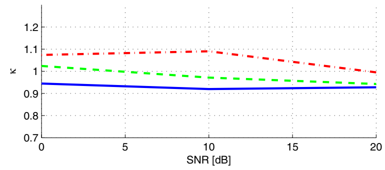

Figure 7. Separation ratio $\tilde{\kappa}$ of linearized estimation model. (a) $A_1 = 1$ (solid), (b) $A_1 = 3$ (dashed), (c) $A_1 = 10$ (dash-dotted), $A_2 = 1$.

$Zp_w(Ze^{j\psi} - a(\boldsymbol{\alpha}))$. Assuming perfect ambiguity resolution, the value $z = Ze^{j\psi}$ is uniquely related to $\hat{\boldsymbol{\alpha}}$ by $Ze^{j\psi} = A_1 e^{j\hat{\alpha}_1} + A_2 e^{j\hat{\alpha}_2}$. We can form the Jacobian of the transformation $J = J_1/J_2$ where $J_1 = -A_1 A_2 \sin(\hat{\alpha}_1 - \hat{\alpha}_2)$, $J_2 = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\hat{\alpha}_1 - \hat{\alpha}_2)}$. The resulting *inner* region PDF is $p_i(\hat{\boldsymbol{\alpha}}|\boldsymbol{\alpha}) = S_1 Z(\hat{\boldsymbol{\alpha}}) p_w(Z(\hat{\boldsymbol{\alpha}}) e^{j\psi(\hat{\boldsymbol{\alpha}})} - a(\boldsymbol{\alpha}))|J|$. The indicator function reflects our assumption of perfect ambiguity resolution. The value $S_1 = S_1(\hat{\alpha}_1, \hat{\alpha}_2) = \mathcal{U}((\hat{\alpha}_2 - \hat{\alpha}_1)_{\bmod 2\pi})\mathcal{U}(\pi - (\hat{\alpha}_2 - \hat{\alpha}_1)_{\bmod 2\pi})$ is unity for solution 1.

VI. Discussion of Results and Conclusions

A numerical evaluation of the region, $\tilde{C}_1, \tilde{C}_2, \tilde{C}_{12}$ and $\tilde{\kappa}$ is shown in Fig 6, 7. The SNR is defined as $\Gamma = \sum_i A_i^2/\sigma_w^2$. We have chosen three sets of A_i values to demonstrate the situation of (a) equal, (b) slightly imbalanced and (c) substantially imbalanced component signal strength. Fig. 8 shows the approximation error of the inner region PDF of the separator output error $p_i(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}|\boldsymbol{\alpha})$. We observe that at higher SNR the error is higher however quite localized. On the other side, at lower SNR the error is lower but affecting wider region. Also the probability of the alternative ambiguity mode becomes higher. This resolution is done by force (the indicator function S_1) which appears as a discontinuity on the PDF error. The impact of the approximation on the separation ratio accuracy itself of any other performance metric (e.g. bit error rate) remains to be open and is a subject of further work.

The information-theoretic analysis of the multiplexing properties of the IWM 2-component CPM phase discriminator showed that the processing has very good separation performance. The numerical evaluation was done under the linearized estimation error assumption which restricts this

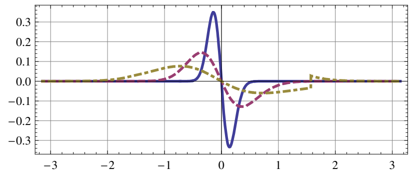

Figure 8. Approximation error of the inner region PDF of the separator output error $p_i(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}|\boldsymbol{\alpha})$ as a function of $\hat{\alpha}_1$ obtained by the linearization approximation and the exact solution. Parameters $\sigma_w \in \{0.2, 0.5, 1\}$ (solid, dashed, dash-dotted), $A_1 = 1, A_2 = 1, \alpha_1 = \pi/2, \alpha_2 = \pi$.

statement to moderate and high SNR region. We evaluated the approximation error of the core PDF with the one obtained by the exact derivation. The separator processing takes samples of the received signal and fully relies on the value domain curved space IWM properties. It does not need any time domain processing. Thus, it can be concatenated with arbitrary outer STCs.

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