Sufficient Statistic and Reduced Dimensionality Equivalent Symbol Space Model for Frequency Flat MIMO Channel with Spatially Nonuniform Delay

Jan Sykora*

Abstract

We first derive an equivalent symbol space discrete channel model for frequency flat block fading MIMO spatial diversity channel with spatially nonuniform path delays. A general nonlinear modulation scheme is assumed by allowing channel symbols to be multidimensional per one antenna. The sufficient symbol spaced statistic dimensionality is shown to be a product of transmit, receive and symbol dimensions unlike for a traditional frequency flat MIMO model where the dimension is a product of receive and symbol dimensions. The traditional model is valid only for spatially uniform delays. We show that the spatially nonuniform path delays cause spatial Inter-Branch Interference (IBI). We introduce a reduced dimensionality equivalent system. The spatial IBI can be reduced using Linear Minimum Mean Square Error (LMMSE) spatial-temporal preprocessing (equalizer). Numerical results show that this dimensionality reducing preprocessing lowers significantly the residual self-noise mean square in comparison with the case ignoring deliberately the higher dimensionality of the sufficient statistic. The spatial processing is found to be the essential one. Additional temporal processing has only small impact on the residual self-noise.

1 Introduction

1.1 Motivation

This paper focuses on the investigation of equivalent symbol space model of MIMO (Multiple Input Multiple Output) spatial diversity communication system with frequency flat fading. The communication system uses wireless propagation medium which is inherently a continuous time waveform medium. However for the purpose of channel capacity investigation, code design, etc., it is useful to work with a discrete time model (see e.g. [1], [2], [3] for some applications). This is done by deriving *equivalent discrete symbol space* model based on the suitable signal expansion and the *sufficient statistic* principle. This somewhat appears to be an overlooked issue in most of the papers which traditionally start with the equivalent discrete model not paying attention to its relation to the underlying continuous time domain channel.

Traditionally the frequency flat MIMO channel model is modeled as Inter-Branch Interference (IBI) free multiplicative distortion memoryless model. In the paper, we will show that this is possible *only* under a *special case* of Spatially Uniform

*The author is with Czech Technical University in Prague, FEE K13137, Technicka 2, 166 27 Praha 6, Czech Republic.
E-mail: Jan.Sykora@fel.cvut.cz. R2.1.0 (21.3.2005)

Delay (SUD) in the propagation paths. Large number of the results on the channel capacity, coding design, etc. rely heavily on the IBI free model of frequency flat MIMO channel. Their properties and functionality is thus directly and substantially influenced by a correctness of this assumption. This paper aims to pointing out those possible problems and gives some remedies to rectify the problems associated with the presence of IBI and with the higher dimensionality of the sufficient statistic of the symbol space model.

A channel with Spatially Nonuniform Delay (SND) can easily appear in the situation when the signals from the individual terminal antennas propagate on the distinct paths or environments (see Fig 1). This can be caused e.g. by antennas with partially directional radiation pattern or by using different polarization in the propagation environment sensitive to the polarization. Such a design of the antenna system can either be induced by the particular system mechanical restrictions or can be intentional since it can help to increase the diversity of the system making the paths more independent especially in the poorly or highly correlated scattering environment.

[Figure 1 about here.]

1.2 Contribution of the paper

The property of SUD/SND is *independent* with the frequency selectivity of individual propagation paths. Channels must be classified by *both* criteria—SUD and per path frequency selectiveness/non-selectiveness. The latter can also be interpreted as the path with non-uniformly/uniformly delayed rays. The SUD can thus be understood as a *generalization* of the frequency non-selectiveness principle *into the spatial dimension*. Our paper shows that the classical IBI free MIMO channel model holds only for the *jointly* SUD and frequency flat channel. We show that the sufficient statistic is inherently multidimensional in the SND case. Any dimensionality reducing preprocessing is therefore only an approximation. To demonstrate this, we develop a LMMSE based spatial-temporal preprocessor.

The paper also addresses the issue of multidimensional signal waveforms allowing proper treatment of *nonlinear* modulations characterized by waveform constellation dimension greater than one.

The idea of necessity of separate channel classification in terms of SUD/SND and frequency selectiveness/non-selectiveness and its consequences on the receiver processing is novel to the best of our knowledge. It is positioned in between general complex frequency selective MIMO channel and classical IBI free (jointly SUD and flat fading) MIMO channel. Both these cases are well known in the technical literature [4]. On one side, the general frequency selective MIMO model is unnecessary complex (with too many degrees of freedom) for our case. On the other side, the one-dimensional observational model of SUD flat fading MIMO channel violates the sufficiency principle.

The LMMSE spatial-temporal dimensionality reducing preprocessor shares common idea with a similar situation of combating with multiple instances of the signal with various delays superposing at receive antennas in a single or multi user environment. This is widely covered in literature (e.g. [5], [6]). Our paper shows that these ideas are also effective in the case of dimensionality reducing processing for SND channel. The resulting performance is better due to having the better a priori knowledge on the observation model which has a lower degree of freedom than the frequency selective case. Dimensionality reduction is particularly important in turbo processing applications (see [7] for the treatment of the frequency selective channel).

1.3 Outline of the paper

In the first part, the paper develops the channel model in the symbol space. It is the *discrete time symbol period* (T_S) spaced model. It is usually obtained by suitable signal expansion. The symbol space model allows, unlike the sampling approach, to express the channel output directly in terms of the channel symbols (codewords). Moreover, we extend the results by considering *arbitrary T_S-fractional delay* in the signal spatial paths and also by considering generally *N_q-multidimensional* spatial branch symbols. This allows us to consider also *nonlinear* modulation schemes unlike the most of other papers. By proper addressing of the issue of multidimensional symbols and arbitrary delays, we will see their influence on the MIMO space-time signal processing and overall properties of the system. The sufficient statistic proves to be $N_T N_R N_q$ dimensional for a general case of mutually unequal path delays. The equivalent model output suffers from IBI caused by mutually unequal T_S-fractional delays.

In the second part of the paper, we derive a *reduced dimensionality equivalent system* with spatial-temporal equalizer. It attempts to turn the sufficient statistic back to N_R dimensional one in the same form as it would appear in the case of spatially uniform delays. This reduces the dimensionality of the channel observation. Finally, the properties of the equalizer are investigated for a random channel matrix and delays. The equalizer is a Linear Minimum Mean Square Error one built on the assumption of perfect Channel State Information (CSI) (channel matrix and delays) assumption. The results can serve as a performance bound for the possibilities of the *linear* spatial-temporal preprocessing.

2 Continuous time system model

Spatial diversity communication system is generally a communication system using a multidimensional channel where the channel dimensionality is physically resolved in spatial dimensions. This is usually achieved by a multi-element antenna arrays having a capability of distinguishing signals to/from various spatial angles.

We will assume a system with N_T dimensional channel input and N_R dimensional output. Such system will be denoted by (N_T, N_R). A traditional description of a general *nonlinearly* modulated signal on the channel input is a N_T dimensional vector signal

$$\mathbf{s}(t) = \sum_n \mathbf{h}'(q'_n, t - nT_S) \tag{1}$$

where q'_n are channel symbols (codewords) and $\mathbf{h}'(q'_n, t)$ are modulation functions describing the expansion part of the modulator. The discrete part of the modulator (the coder) is described by the output equation

$$q'_n = q'(d_n, \sigma_n), \quad q'_n \in \left\{ q'^{(i)} \right\}_{i=1}^{M_q} \tag{2}$$

where $d_n \in \{d^{(i)}\}_{i=1}^{M_d}$ are data symbols and $\sigma_n \in \{\sigma^{(i)}\}_{i=1}^{M_\sigma}$ are the modulator states. The modulator states are ruled by the state equation

$$\sigma_{n+1} = \sigma(d_n, \sigma_n). \tag{3}$$

An arbitrary modulated signal (including nonlinear modulations) can be expressed with a linear (however generally multidimensional) expansion part (for details see [8] or [9], [10]). Thus we can write

$$\mathbf{s}(t) = \sum_n \mathbf{Q}_n^T \mathbf{h}(t - nT_S) \tag{4}$$

where

$$\mathbf{Q}_n = [\mathbf{q}_{n,1}, \ldots, \mathbf{q}_{n,N_T}] = \mathbf{Q}(d_n, \sigma_n) \in \left\{\mathbf{Q}^{(i)}\right\}_{i=1}^{M_q} \tag{5}$$

is $N_q \times N_T$ space-time channel symbol (codeword). N_q is a dimension of the channel symbol in one spatial branch. In a special case of *linear modulation*, it becomes $N_q = 1$. Impulse $\mathbf{h}(t)$ is N_q dimensional modulation impulse which is assumed to be shared by all spatial dimensions. The impulse is further assumed to be a complex *Nyquist* one with *orthonormal components* (see [8]). Its energy time-domain correlation function is

$$\mathcal{R}_{\mathbf{h}}^{\mathcal{E}}[m] = \mathcal{R}_{\mathbf{h}}^{\mathcal{E}}(mT_S) = \delta[m]\mathbf{I}. \tag{6}$$

The modulated signal passes through (N_T, N_R) channel. The k-th element of the channel output vector $\mathbf{x}$ is a superposition of contributions from all channel inputs with additive white Gaussian noise (AWGN)

$$\begin{aligned} x_k(t) &= u_k(t) + w_k(t) \\ &= \sum_{i=1}^{N_T} \mathcal{g}_{ki}[s_i(t)] + w_k(t). \end{aligned} \tag{7}$$

Signal $u_k(t)$ is the useful signal at k-th receiver branch. Complex noise components in individual branches are assumed to be IID circularly symmetric Gaussian processes with correlation function $R_{w_k}(\tau) = 2N_0\delta(\tau)$. An individual contribution of the signal from i-th input to k-th output is generally described by the operator $\mathcal{g}_{ki}[\bullet]$. In a special case of *frequency flat linear* channel this operator takes a form of

$$\mathcal{g}_{ki}[s_i(t)] = g_{ki} s_i(t - \tau_{ki}) \tag{8}$$

where g_{ki} are channel transfer coefficients and τ_{ki} are path delays. We also define a matrix $[\mathbf{G}]_{ki} = g_{ki}$. We consider a block-constant fading channel which has coefficients constant during the channel observation (typically a data block). This will be assumed to hold in the following treatment.

The MIMO channel is said to be *Spatially Uniform Delay (SUD)* channel if $\tau_{ki} = \tau_{k'i'}$, $\forall k, k', i, i'$. In the opposite case, it is said to be generally *Spatially Nonuniform Delay (SND)* channel. We also define special cases. The channel is *Spatially Transmit-Uniform Delay (STUD)* if $\tau_{ki} = \tau_{ki'}$, $\forall k, i, i'$ and *Spatially Receive-Uniform Delay (SRUD)* if $\tau_{ki} = \tau_{k'i}$, $\forall k, k', i$.

3 Equivalent symbol space channel model

Up to this point, we treated the channel in the continuous time domain. However it is very useful to find its equivalent symbol space model. The symbol space model is discrete time symbol period spaced model. Its input are directly the channel symbols and the output is symbol spaced discrete time observation as a function of channel symbols. This model uses a suitably defined signal expansions. If it is possible, it is beneficial to use orthonormal expansion (signal space expansion). The orthonormality assumption guarantees the mutual numerical equivalence of the inner product operation in both domains. The signal space (contrary to the sampling) expansion is necessary in order to relate the equivalent model directly to the channel symbols (codewords).

The situation on the transmitter side is easy to handle since we assumed Nyquist modulation impulses with orthonormal components. Therefore the symbol space expansion for n-th symbol is easily recognized as $\mathbf{s}_n = \mathbf{Q}_n$ with the expansion basis $\mathbf{h}(t - nT_S)$. Because of the Nyquist modulation impulse assumption, the whole signal expansion $\mathbf{s} = [\ldots, \mathbf{s}_n, \ldots]$ has orthonormal basis $\mathcal{A} = \{\mathbf{h}(t - nT_S)\}_n$.

Contrary to the transmitter side, the situation at the receiver side is somewhat more complicated. This is because of the channel parameters. Particularly we allowed arbitrary delays τ_{ki} on individual branches. The expansion basis and the corresponding signal with a simple relation to transmitted codewords (it does not have to be directly the received signal) cannot be identified by a similar simplistic approach. We need to adopt a systematic procedure based on the information theory—namely using the principle of sufficient statistic. For a good background see e.g. [11] or [12].

We will assume that the receiver has a *perfect knowledge* of the CSI, i.e. the gains g_{ki} and delays τ_{ki}. A conditional probability density function (PDF) of the received signal conditioned by the transmitted signal is

$$\begin{aligned} p(\mathbf{x}|\mathbf{s}) &= c e^{-\frac{1}{2N_0}\sum_{k=1}^{N_R}\int_{-\infty}^{\infty}|x_k - u_k(\mathbf{s})|^2\,dt} \\ &= c e^{-\frac{1}{2N_0}\sum_{k=1}^{N_R}\int_{-\infty}^{\infty}|x_k|^2 - 2\Re\left[x_k u_k^*(\mathbf{s})\right] + |u_k(\mathbf{s})|^2\,dt}. \end{aligned} \tag{9}$$

The sufficient statistic is found (based on the Neyman-Fisher factorization theorem) inside the inner product evaluation

$$\begin{aligned} &\sum_{k=1}^{N_R}\int_{-\infty}^{\infty} x_k u_k^*\,dt = \\ &= \sum_{k=1}^{N_R}\int_{-\infty}^{\infty} x_k \sum_{i=1}^{N_T} g_{ki}^* \sum_n \mathbf{q}_{n,i}^H \mathbf{h}^*(t - \tau_{ki} - nT_S)\,dt. \end{aligned} \tag{10}$$

Clearly, on condition of perfect CSI knowledge at receiver the sufficient statistic is

$$\begin{aligned} \mathbf{y}_{n,k,i} = \mathbf{T}_k(\mathbf{x}) = \int_{-\infty}^{\infty} x_k \mathbf{h}^*(t - \tau_{ki} - nT_S)\,dt, \\ i \in \{1, \ldots, N_T\},\ k \in \{1, \ldots, N_R\}. \end{aligned} \tag{11}$$

This symbol spaced observations form the output of symbol space model.

The expansion basis for the received signal (matched filter bank) is

$$\mathcal{B} = \{\mathcal{B}_n\}_n = \left\{\{\mathbf{h}(t - \tau_{ki} - nT_S)\}_{k,i}\right\}_n. \tag{12}$$

Figure 2 shows a graphical representation of the sufficient statistic evaluation. The cross-correlation part of the receiver detector metric is then get from the sufficient statistic

$$\sum_{k=1}^{N_R} \int_{-\infty}^{\infty} x_k u_k^* \, dt = \sum_{k=1}^{N_R} \sum_{i=1}^{N_T} g_{ki}^* \sum_n \mathbf{q}_{n,i}^H \mathbf{y}_{n,k,i}. \tag{13}$$

The basis $\mathcal{B}$ is generally nonorthogonal. It becomes *orthonormal* only in the case of *SUD* channel (i.e. $\tau_{ki} = \text{const}$).

[Figure 2 about here.]

Based on this statistic, we can equivalently investigate the *symbol space channel* with the input for n-th symbol

$$\mathbf{s}_n = \mathbf{Q}_n = [\mathbf{q}_{n,1}, \ldots, \mathbf{q}_{n,N_T}] \tag{14}$$

and the output $\mathbf{y}_{n,k,i}$ related to the received signal $x_k(t)$ by the equation (11). It is important to stress that the perfect CSI knowledge was necessary.

The properties analysis of the new equivalent symbol space channel output will reveal its relation to the transmitted signal and the channel gains—which was the goal of our effort. We substitute for the received signal to get

$$\begin{aligned} \mathbf{y}_{n,k,i} &= \\ &= \int_{-\infty}^{\infty} \left(\sum_{i'=1}^{N_T} g_{ki'} \sum_{n'} \mathbf{q}_{n',i'}^T \mathbf{h}(t - \tau_{ki'} - n'T_S) + w_k(t) \right) \\ &\quad \times \mathbf{h}^*(t - \tau_{ki} - nT_S) \, dt. \end{aligned} \tag{15}$$

Skipping some algebra we get the equivalent symbol space system model

$$\begin{aligned} \mathbf{y}_{n,k,i} \;\; = \;\; & \sum_{i'=1}^{N_T} g_{ki'} \sum_{n'} \mathcal{R}_{\mathbf{h}}^{\mathcal{E}*} \left((n' - n)T_S + \tau_{ki'} - \tau_{ki} \right) \mathbf{q}_{n',i'} \\ & + \mathbf{z}_{n,k,i} \end{aligned} \tag{16}$$

where the impulse energy correlation matrix is $\mathcal{R}_{\mathbf{h}}^{\mathcal{E}}(\tau) = \int_{-\infty}^{\infty} \mathbf{h}(t + \tau)\mathbf{h}^H(t) \, dt$. The noise component can be easily verified to be complex circularly symmetric Gaussian with $(N_q \times N_q)$ covariance matrix

$$\mathbf{C}_{\mathbf{z}_{n,k,i}} = \sigma_z^2 \mathbf{I} = 2N_0 \mathbf{I}. \tag{17}$$

4 Spatially non-uniform delay and inter-branch interference

In a general case of arbitrary channel that is *not STUD channel*, the expression for $\mathbf{y}_{n,k,i}$ suffers from the inter-symbol interference from all transmitter branches—*Inter-Branch Interference (IBI)*. It is interesting to notice that only transmit uniformity STUD is needed to avoid IBI. The receive non-uniformity is accommodated harmlessly by proper delay estimator at individual receiver branches. IBI is a mixture of spatial and also temporal domain interfering symbols. Amount of the interfering symbols increases with the dimensionality of the MIMO system and therefore have bigger impact then in the case of SISO system. As we observe, the problem is caused by existence of T_S-fractional delay differences, which is something that we have to consider in the real system deployment. It is important to stress that the individual branch channels are frequency flat and all interference is purely caused by the nonuniform path delays even if the perfect CSI is available at receiver. In the case of frequency selective channel, additional interference from delayed branch signal replicas would appear. This fact is elementary and the necessity of the equalization (unlike for a frequency flat fading) is widely recognized. Unfortunately, the consequences of IBI in frequency *flat* fading case and efficient counter measures are still far from being well understood and they are widely ignored. For some results mapping possible consequences see e.g. [13].

On the other side, in the case of at least STUD channel, i.e. $\tau_{ki} = \tau_{ki'}$, $\forall k, i, i'$, we get using the properties of modulation impulse

$$\mathbf{y}_{n,k,i} = \sum_{i'=1}^{N_T} g_{ki'} \mathbf{q}_{n,i'} + \mathbf{z}_{n,k,i}\,. \tag{18}$$

The sufficient statistic is now identical for all transmitters (index i). We can simplify previous results into a simple matrix notation (dropping now superfluous dependence on i and by stacking the vectors)

$$\mathbf{y}_n = \mathbf{G}_{N_q} \mathbf{q}_n + \mathbf{z}_n \tag{19}$$

where the stacked vectors are $\mathbf{y}_n = [\mathbf{y}_{n,1}^T, \ldots, \mathbf{y}_{n,N_R}^T]^T$, $\mathbf{q}_n = [\mathbf{q}_{n,1}^T, \ldots, \mathbf{q}_{n,N_T}^T]^T$, $\mathbf{z}_n = [\mathbf{z}_{n,1}^T, \ldots, \mathbf{z}_{n,N_R}^T]^T$ and the stacked channel matrix is get by the Kronecker matrix product with $(N_q \times N_q)$ identity matrix $\mathbf{G}_{N_q} = \mathbf{G} \otimes \mathbf{I}_{N_q}$. The system equivalent signal space model now corresponds with the one which is traditionally used for flat fading MIMO channel with an extension for multidimensional channel symbols per antenna. We see that this is possible *only* for transmit uniform delays. It is also very useful to realize that the dimensionality of individual transmitter branches (which is greater than one for nonlinear or block coded modulations) in fact multiplies with the spatial dimensionality to the overall dimensionality of the equivalent channel.

From the results above, we can conclude that there exists the discrete symbol space equivalent representation of general MIMO system with multidimensional symbols per branch. *Multidimensionality* of the branch symbol allows the model to be used for general nonlinear and block coded modulations. The output of this discrete system corresponds to the output signal $y_k(t)$ and forms the sufficient statistic. Perfect CSI knowledge is required. A necessary condition for avoiding ISI and IBI is the modulation impulse being Nyquist one with orthonormal components and spatially transmit-uniform delay.

5 Reduced dimensionality symbol space system with spatial-temporal preprocessing

A high dimensionality of the sufficient statistic in the general SND channel in comparison with the SUD case is an obvious problem. We can attack the problem either by *ignoring* it or by a *spatial-temporal preprocessing (equalization)*. Both approaches reduce the symbol space system output dimensionality into the one corresponding to the SUD channel (i.e. $N_R N_q$ per one symbol slot). The first approach, picking only one of the outputs per receive antenna and ignoring others, will suffer the IBI self-noise in full. The second approach with dimensionality reducing ($N_T N_R N_q$ to $N_R N_q$) spatial-temporal equalizer can reduce the IBI while reducing the dimensionality for the subsequent processing. The reduced dimensionality symbol space system is on Fig. 3.

[Figure 3 about here.]

In this section, we build the Linear Minimum Mean Square Error (LMMSE) spatial-temporal non-recursive equalizer which will attempt to minimize residual IBI in reduced dimensionality system for frequency flat fading channel with transmit nonuniform T_S-fractional delays. The equalizer will attempt to convert this channel back into the case with the uniform delays and therefore with no IBI. This reduces the dimensionality of the output by factor N_T. The derivation will be carried out for a *special case of linear modulation,* i.e. $N_q = 1$.

5.1 Equivalent system model for linear modulation

Now we develop a compact matrix description of the system. The general symbol space equivalent system model based on the sufficient statistic is given by (16). For the special case of linear modulation ($N_q = 1$) we get

$$\begin{aligned} y_{n,k,i} &= \sum_{i'=1}^{N_T} g_{ki'} \sum_{n'} \mathcal{R}_h^{g*}\left((n'-n)T_S + \tau_{ki'} - \tau_{ki}\right) q_{n',i'} \\ &\quad + z_{n,k,i} \end{aligned} \tag{20}$$

and

$$\mathrm{var}[z_{n,k,i}] = \sigma_z^2 = 2N_0. \tag{21}$$

We will restrict the next treatment to the *sliding block non-causal* signal processing. This is a typical situation in packet oriented communication systems where the signal of the whole packet is stored in the memory and the signal processing is carried out afterwards. In our notation, it means that $n, n' \in [-L, L]$ where L is the depth of history or future available for processing channel symbols at zero time index[1] $q_{0,i}$. The L value is set according to the impulse correlation $\mathcal{R}_h^g(t)$ function in such a way that all significantly nonzero values are included in the summation, i.e. $\mathcal{R}_h^g(t) \approx 0$ for all $t : |t| > LT_S$.

In this section, we develop a simple vector notation for the overall spatial-temporal equivalent model that later allows a straightforward application of the LMMSE equalizer principle. The channel symbols are assumed to be stationary. Performance evaluation of the equalizer can be thus investigated with respect to the symbols $q_{n,i}$ for $n = 0$. This will be assumed for the rest of the paper. We denote virtual transfer coefficients

$$b_{n',i'}^{(n,k,i)} = g_{ki'} \mathcal{R}_h^{g*}\left((n'-n)T_S + \tau_{ki'} - \tau_{ki}\right), \tag{22}$$

$$\mathbf{b}_{n'}^{(n,k,i)} = [b_{n',1}^{(n,k,i)}, \ldots, b_{n',N_T}^{(n,k,i)}]^T \tag{23}$$

and $((2L+1)N_T \times 1)$ stacked vector

$$\tilde{\mathbf{b}}^{(n,k,i)} = \left[\mathbf{b}_{-L}^{(n,k,i)T}, \ldots, \mathbf{b}_{L}^{(n,k,i)T}\right]^T. \tag{24}$$

[1] We assume stationary sequence of channel symbols.

We use similar notation for $((2L+1)N_T \times 1)$ stacked channel symbols

$$\tilde{\mathbf{q}} = \begin{bmatrix} [q_{-L,1}, \ldots, q_{-L,N_T}]^T \\ \vdots \\ [q_{L,1}, \ldots, q_{L,N_T}]^T \end{bmatrix}. \tag{25}$$

Then the channel output relevant to the channel symbols $q_{0,i}$ is

$$\begin{aligned} y_{n,k,i} &= \sum_{i'=1}^{N_T} \sum_{n'=-L}^{L} b_{n',i'}^{(n,k,i)} q_{n',i'} + z_{n,k,i} \\ &= \tilde{\mathbf{b}}^{(n,k,i)T} \tilde{\mathbf{q}} + z_{n,k,i}, \quad n = -L_e, \ldots, L_e \end{aligned} \tag{26}$$

where the *equalizer filter temporal processing window length* is $L_e \leq L$. These observations ignore all that do not depend on the $q_{0,i}$ symbol. The parameter L_e is introduced for intentional shortening of the equalizer temporal processing length. It can reduce the computational complexity but with the price paid by possibly increased residual self-noise.

Values $y_{k,i}$ and $z_{k,i}$ are similarly stacked into $((2L_e+1)N_T \times 1)$ vectors

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} [y_{-L_e,k,1}, \ldots, y_{-L_e,k,N_T}]^T \\ \vdots \\ [y_{L_e,k,1}, \ldots, y_{L_e,k,N_T}]^T \end{bmatrix}, \tag{27}$$

$$\tilde{\mathbf{z}}_k = \begin{bmatrix} [z_{-L_e,k,1}, \ldots, z_{-L_e,k,N_T}]^T \\ \vdots \\ [z_{L_e,k,1}, \ldots, z_{L_e,k,N_T}]^T \end{bmatrix}. \tag{28}$$

We also construct a $(N_T \times (2L+1)N_T)$ matrix

$$\mathbf{B}_{n,k} = \begin{bmatrix} \tilde{\mathbf{b}}^{(n,k,1)T} \\ \vdots \\ \tilde{\mathbf{b}}^{(n,k,N_T)T} \end{bmatrix} \tag{29}$$

and $((2L_e+1)N_T \times (2L+1)N_T)$ stacked matrix

$$\tilde{\mathbf{B}}_k = \begin{bmatrix} \mathbf{B}_{-L_e,k} \\ \vdots \\ \mathbf{B}_{L_e,k} \end{bmatrix}. \tag{30}$$

Now, we can simply express the equivalent model at k-th receiver including the temporal dependencies

$$\tilde{\mathbf{y}}_k = \tilde{\mathbf{B}}_k \tilde{\mathbf{q}} + \tilde{\mathbf{z}}_k. \tag{31}$$

The next step is to combine all receiver branches into one large vector/matrix system which describes all spatial and temporal dependences in the equivalent system. We define $((2L_e+1)N_T N_R \times 1)$ stacked vectors

$$\tilde{\mathbf{y}} = \left[\tilde{\mathbf{y}}_1^T, \ldots, \tilde{\mathbf{y}}_{N_R}^T\right]^T, \tag{32}$$

$$\tilde{\mathbf{z}} = \left[\tilde{\mathbf{z}}_1^T, \ldots, \tilde{\mathbf{z}}_{N_R}^T\right]^T \tag{33}$$

and large $((2L_e+1)N_T N_R \times (2L+1)N_T)$ stacked matrix

$$\tilde{\mathbf{B}} = \begin{bmatrix} \tilde{\mathbf{B}}_1 \\ \vdots \\ \tilde{\mathbf{B}}_{N_R} \end{bmatrix}. \tag{34}$$

The overall *spatial-temporal symbol space equivalent model* in simple matrix notation is then

$$\tilde{\mathbf{y}} = \tilde{\mathbf{B}}\tilde{\mathbf{q}} + \tilde{\mathbf{z}}. \tag{35}$$

A graphical demonstration of the vector and matrix stacking in the model is shown on Fig. 4.

[Figure 4 about here.]

5.2 Linear Minimum MSE equalizer

The overall spatial-temporal equivalent matrix model (35) allows very easy application of the LMMSE principle. Our goal will be minimization of Mean Square Error (MSE) over all receiver branches between the equalizer output and the desired value. The *desired* value is the equivalent channel model output that we would obtain if the channel was *STUD*. It has dimensionality N_R (one output per receive antenna). The desired equalizer output at time instant $n=0$ is $(N_R \times 1)$ vector

$$\boldsymbol{\theta} = \begin{bmatrix} \sum_{i=1}^{N_T} g_{1i} q_{0,i} \\ \vdots \\ \sum_{i=1}^{N_T} g_{N_R i} q_{0,i} \end{bmatrix}. \tag{36}$$

This can be easily get in the matrix form from the stacked vector $\tilde{\mathbf{q}}$ by

$$\boldsymbol{\theta} = \tilde{\mathbf{G}}\tilde{\mathbf{q}} \tag{37}$$

where the stacked $(N_R \times (2L+1)N_T$ channel matrix is ($\mathbf{O}$ is $(N_R \times L N_T)$ zero matrix)

$$\tilde{\mathbf{G}} = [\mathbf{O}, \mathbf{G}, \mathbf{O}]. \tag{38}$$

The LMMSE estimate for the time instant $n=0$ is sought in a linear non-recursive form

$$\hat{\boldsymbol{\theta}} = \hat{\mathbf{V}}\tilde{\mathbf{y}} \tag{39}$$

where $\hat{\mathbf{V}}$ is $(N_R \times (2L_e + 1)N_T N_R)$ equalizer spatial-temporal filter matrix. The estimate minimizes the MSE error

$$\begin{aligned} \hat{\mathbf{V}} &= \arg\min_{\check{\mathbf{V}}} \|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|^2 \\ &= \arg\min_{\check{\mathbf{V}}} \left\| \check{\mathbf{V}}\tilde{\mathbf{y}} - \tilde{\mathbf{G}}\tilde{\mathbf{q}} \right\|^2 . \end{aligned} \tag{40}$$

This MSE can be also interpreted as a *residual Self-Noise Mean Square (SNMS)* value.

The standard LMMSE solution (see e.g. [11]) is

$$\hat{\mathbf{V}} = \mathbf{R}_{\boldsymbol{\theta},\tilde{\mathbf{y}}} \mathbf{R}_{\tilde{\mathbf{y}}}^{-1} \tag{41}$$

where the correlation matrices can be easily get as

$$\begin{aligned} \mathbf{R}_{\boldsymbol{\theta},\tilde{\mathbf{y}}} &= \mathrm{E}[\boldsymbol{\theta}\tilde{\mathbf{y}}^H] \\ &= \tilde{\mathbf{G}}\mathbf{R}_{\tilde{\mathbf{q}}}\tilde{\mathbf{B}}^H \end{aligned} \tag{42}$$

and

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{y}}} &= \mathrm{E}[\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H] \\ &= \tilde{\mathbf{B}}\mathbf{R}_{\tilde{\mathbf{q}}}\tilde{\mathbf{B}}^H + \mathbf{R}_{\tilde{\mathbf{z}}}. \end{aligned} \tag{43}$$

The expectation is carried out over the random variables that are finite observation ergodic[2] in the observation window. Noise correlation $(N_R(2L_e + 1)N_T \times N_R(2L_e + 1)N_T)$ matrix $\mathbf{R}_{\tilde{\mathbf{z}}}$ has block-wise diagonal structure

$$\mathbf{R}_{\tilde{\mathbf{z}}} = \begin{bmatrix} \mathbf{R}_{\tilde{\mathbf{z}}_1} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \ddots & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{R}_{\tilde{\mathbf{z}}_{N_R}} \end{bmatrix} \tag{44}$$

where diagonal matrices contain modulation impulse function correlation values

$$\begin{aligned} &[\mathbf{R}_{\tilde{\mathbf{z}}_k}]_{nN_T+i,n'N_T+i'} = \\ &\quad = 2N_0 \int_{-\infty}^{\infty} h^*(t - \tau_{ki} - nT_S) h(t - \tau_{ki'} - n'T_S)\, dt \\ &\quad = 2N_0 \mathcal{R}_h^{\varepsilon *}\left((n' - n)T_S + \tau_{ki'} - \tau_{ki}\right). \end{aligned} \tag{45}$$

5.3 Equivalent model of the reduced dimensionality system

The equalizer provides on its output a new equivalent reduced dimensionality system output. This system has now only N_R dimensional output per symbol period—in principle the same as if the channel delays were mutually equivalent. The equalized

[2] They demonstrate all their randomness within the observation window.

equivalent system model is

$$\hat{\boldsymbol{\theta}} = \hat{\mathbf{V}}\tilde{\mathbf{y}} = \mathbf{A}\tilde{\mathbf{q}} + \boldsymbol{\xi} \tag{46}$$

where the new system $(N_R \times (2L+1)N_T)$ matrix is $\mathbf{A} = \hat{\mathbf{V}}\tilde{\mathbf{B}}$ and the new noise is $\boldsymbol{\xi} = \hat{\mathbf{V}}\tilde{\mathbf{z}}$ with covariance matrix

$$\mathbf{R}_{\boldsymbol{\xi}} = \hat{\mathbf{V}}\mathbf{R}_{\tilde{\mathbf{z}}}\hat{\mathbf{V}}^H. \tag{47}$$

Generally, the noise is no longer white.

5.4 Properties of the reduced dimensionality system—equalizer performance

The equalizer performance can be assessed by the evaluation of the residual self-noise (SNMS) at the equalizer output. It is defined as

$$\boldsymbol{\theta}_\epsilon = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta} \tag{48}$$

where $\boldsymbol{\theta}$ is the ideal desired value of the STUD channel. The MSE (SNMS) matrix can be get (see [11])

$$\begin{aligned} \mathbf{R}_{\boldsymbol{\theta}_\epsilon} &= \mathrm{E}[\boldsymbol{\theta}_\epsilon \boldsymbol{\theta}_\epsilon^H] \\ &= \mathbf{R}_{\boldsymbol{\theta}} - \mathbf{R}_{\boldsymbol{\theta},\tilde{\mathbf{y}}}\mathbf{R}_{\tilde{\mathbf{y}}}^{-1}\mathbf{R}_{\tilde{\mathbf{y}},\boldsymbol{\theta}} \\ &= \tilde{\mathbf{G}}\mathbf{R}_{\tilde{\mathbf{q}}}\tilde{\mathbf{G}}^H - \hat{\mathbf{V}}\tilde{\mathbf{B}}\mathbf{R}_{\tilde{\mathbf{q}}}^H\tilde{\mathbf{G}}^H. \end{aligned} \tag{49}$$

The overall SNMS for LMMSE equalizer is then

$$\mathrm{mse}[\hat{\boldsymbol{\theta}}] = \|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|^2 = \mathrm{tr}(\mathbf{R}_{\boldsymbol{\theta}_\epsilon}). \tag{50}$$

The overall SNMS of the LMMSE on its own has however only a limited information value as a measure of the equalizer advantage over the *unequalized* system. Therefore we next compare this SNMS to the SNMS that would be present if there was no equalizer. An actual meaning of the output of an unequalized system with N_R dimensional output is of course rather ad-hoc. This is because we know that a theoretically correct is $N_R N_T$ dimensional sufficient statistic. Anything else is a violation of the sufficient statistic. However we can consider as an *ad-hoc reference unequalized system* the one with N_R dimensional output (the same as for mutually equal path delays). This will provide an indication what happens if we intentionally ignore the sufficient statistic. For the unequalized system, we choose ad-hoc on each receiver *only one* (ignoring other) of the components from $\{y_{0,k,i}\}_{i=1}^{N_T}$, particularly we decide for $i = 1$. This choice cannot affect the comparison in terms of mean long time scale performance much since the path delays are all random over individual frames.

The unequalized output is

$$\psi_k = y_{0,k,1}, \quad k \in \{1, \ldots, N_R\}. \tag{51}$$

This can be written in matrix notation as

$$\boldsymbol{\psi} = \mathbf{U}\tilde{\mathbf{y}} \tag{52}$$

where $(N_R \times (2L_e+1)N_T N_R)$ matrix $\mathbf{U}$ is all zero matrix except for the components with index corresponding to $y_{0,k,1}$

$$[\mathbf{U}]_{k,i} = \begin{cases} 1; & i = (k-1)N_T(2L_e+1) + L_e N_T + 1 \\ 0; & \text{elsewhere} \end{cases} \tag{53}$$

where $k \in \{1, \ldots, N_R\}$, $i \in \{1, \ldots, (2L_e+1)N_T N_R\}$. The error vector is

$$\begin{aligned} \boldsymbol{\zeta} &= \boldsymbol{\psi} - \boldsymbol{\theta} \\ &= \mathbf{U}\tilde{\mathbf{y}} - \tilde{\mathbf{G}}\tilde{\mathbf{q}} \\ &= \mathbf{U}(\tilde{\mathbf{B}}\tilde{\mathbf{q}} + \tilde{\mathbf{z}}) - \tilde{\mathbf{G}}\tilde{\mathbf{q}}. \end{aligned} \tag{54}$$

The MSE matrix of unequalized output is

$$\begin{aligned} \mathbf{R}_{\boldsymbol{\zeta}} &= \mathrm{E}[\boldsymbol{\zeta}\boldsymbol{\zeta}^H] \\ &= \mathbf{U}\tilde{\mathbf{B}}\mathbf{R}_{\tilde{\mathbf{q}}}\tilde{\mathbf{B}}^H\mathbf{U}^H + \mathbf{U}\mathbf{R}_{\tilde{\mathbf{z}}}\mathbf{U}^H + \tilde{\mathbf{G}}\mathbf{R}_{\tilde{\mathbf{q}}}\tilde{\mathbf{G}}^H \\ &\quad -\mathbf{U}\tilde{\mathbf{B}}\mathbf{R}_{\tilde{\mathbf{q}}}\tilde{\mathbf{G}}^H - \tilde{\mathbf{G}}\mathbf{R}_{\tilde{\mathbf{q}}}\tilde{\mathbf{B}}^H\mathbf{U}^H \end{aligned} \tag{55}$$

and the overall MSE of unequalized output is

$$\mathrm{mse}[\boldsymbol{\psi}] = \|\boldsymbol{\psi} - \boldsymbol{\theta}\|^2 = \mathrm{tr}(\mathbf{R}_{\boldsymbol{\zeta}}). \tag{56}$$

6 Numerical results

In this section, we present numerical results. We evaluated the above derived equalizer performance for random values of $\mathbf{G}$ and τ_{ki}. These values are constant within one equalizer observation frame. The overall *average self-noise mean square* performance results were get by a numerical averaging over a random complex Gaussian channel matrix $\mathbf{G}$ and random uniformly distributed delays $\tau_{ki} \sim \mathcal{UD}(\tau_{\min}, \tau_{\max})$. This corresponds to an average performance for large number of transmitted blocks in block fading channel. We assumed $\tau_{\min} = -T_S/2$ and $\tau_{\max} = T_S/2$. In all cases, we assumed uncorrelated channel symbols with unity mean square $\mathbf{R}_{\tilde{\mathbf{q}}} = \mathbf{I}$. The signal to noise ratio per one receiver is defined to be

$$\gamma = \frac{N_T \; \mathrm{E}[|q_{n,i}|^2]}{N_0}. \tag{57}$$

The channel matrix $\mathbf{G}$ is assumed to have IID Rayleigh components with unity variance. The modulation impulse is assumed to be Root Raised Cosine (RRC) with the roll-off $\alpha = 0.6$. The length of modulation impulse correlation with significant influence is $L = 3$.

Figures 5, 6, and 7 show the results comparing the SNMS of reduced dimensionality (N_R per one symbol slot) model output for two cases. The first is the unequalized one deliberately ignoring multidimensional sufficient statistic. The second case is the one with linear spatial-temporal dimensionality reducing preprocessing (equalization) with various degrees of temporal processing (parameter L_e). The case of $L_e = 0$ corresponds to spatial-only (no temporal processing). Observing the corresponding results, we see that *spatial* processing is the *essential* one. The additional temporal processing ($L_e \geq 1$) gives only a negligible gain in terms of residual self-noise. The statement is applicable to the average (large number of independent blocks) SNMS. An interpretation of the influence on the performance within the given block is still an open question.

[Figure 5 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

7 Conclusions

We derived an equivalent symbol space channel model for *frequency flat spatially nonuniform delay* MIMO fading channel. The model is based on the sufficient statistic principle. The symbol space model is a discrete time symbol spaced observation directly depending the channel symbols (codewords) at the input. Multidimensional (per antenna) channel symbols are considered. This allows to use the results also for *nonlinear* modulation schemes. It is shown that the sufficient statistic must be $N_T N_R N_q$ dimensional per one symbol slot in a general case. The traditional IBI free multiplicative channel symbol spaced model with N_R dimensional output is proved to be inadequate for the spatially nonuniform delay channel.

The equivalent channel output is heavily affected by the spatial IBI in the case of spatially nonuniform T_S-fractional path delay channel. We show that for a linear modulation this highly dimensional statistic can be converted into reduced dimensionality observation by means of spatial-temporal equalization. The reduced dimensionality is equal to the traditionally considered IBI free N_R dimensional output symbol space model. There is derived linear minimum MSE equalizer that reduces the IBI. However the new equivalent system model, as observed at the equalizer output, have generally correlated Gaussian noise. Average self-noise mean square performance of the equalizer for random channel matrix and path delays was investigated. The equalizer assumed the perfect CSI knowledge (gains, delays). It provides an IBI canceling performance *bound* in the MSE sense on the *linear* spatial-temporal preprocessing.

The results of the paper rise considerable doubts of the vitality of the widely used MIMO frequency flat memoryless IBI free fading channel model without properly observing spatial uniformity of path delays. The T_S-fractional path delays differences destroy the original Nyquist impulse orthogonality. The presence of these T_S-fractional delays is practically unavoidable in any practical system with partially directive antennas or other spatially selective properties of individual antennas. A possible linear spatial-temporal preprocessing can help to reduce the IBI problem while reducing the dimensionality of the observation to N_R. Numerical results show that this dimensionality reducing preprocessing lowers significantly the residual MSE in comparison with the case ignoring deliberately the higher dimensionality of the sufficient statistic. We observed that the *spatial* processing is the *essential* one. Additional temporal processing has only small impact on average (large number of independent blocks) residual self-noise. The price we pay for spatial-temporal processing reducing the output dimensionality is in the lost noise whiteness and a considerably increased processing load. The processing complexity and residual IBI self-noise easily erases all elegance of some MIMO decoding technique (e.g. Alamouti code) originally devised for N_R dimensional IBI free symbol space channel model and therefore a care must be taken when they are applied in SND channel.

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List of Figures

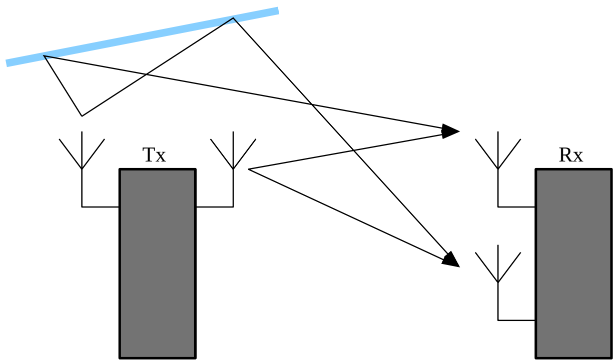

Figure 1: An example of the (2,2) MIMO channel with spatially nonuniform delay.

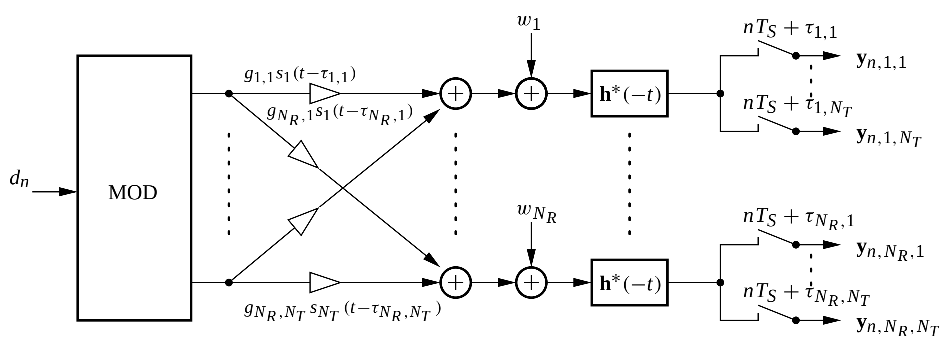

Figure 2: Sufficient statistic for a frequency flat spatially nonuniform delay channel.

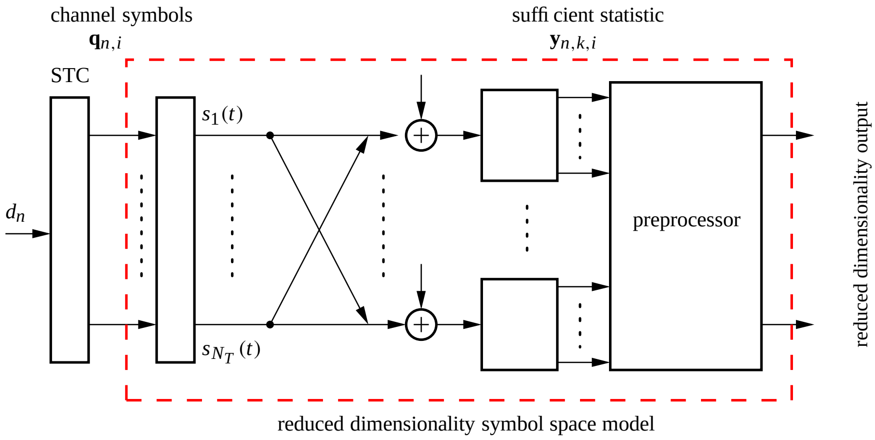

Figure 3: Reduced dimensionality symbol space system.

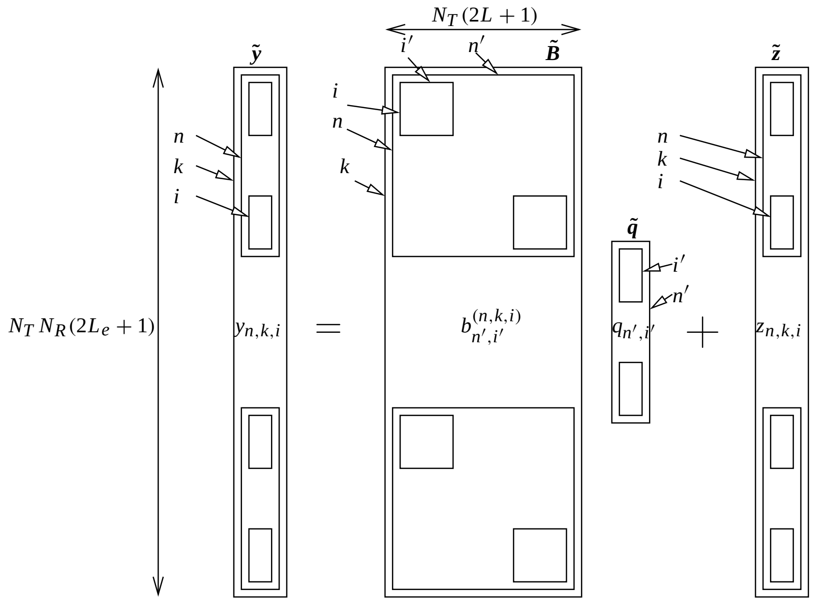

Figure 4: Vector and matrix stacking in the symbol space equivalent model for $N_q = 1$.

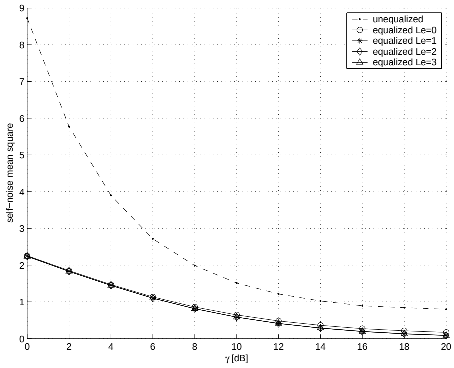

Figure 5: A comparison of average self-noise mean square for equalized and unequalized reduced dimensionality model for $N_T = 2,\ N_R = 2$.

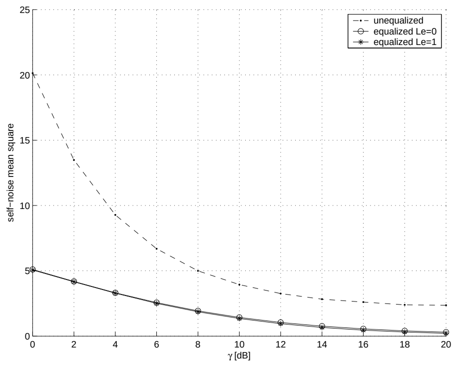

Figure 6: A comparison of average self-noise mean square for equalized and unequalized reduced dimensionality model for $N_T = 3,\ N_R = 3$.

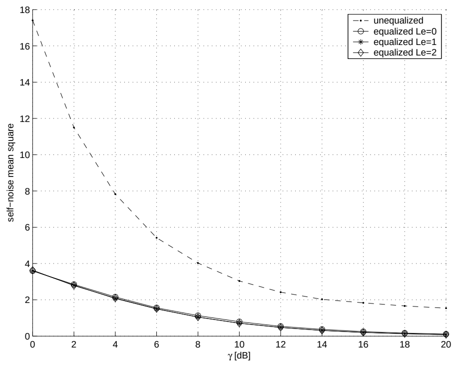

Figure 7: A comparison of average self-noise mean square for equalized and unequalized reduced dimensionality model for $N_T = 2,\ N_R = 4$.