Receiver Constellation Waveform Subspace Preprocessing for Burst Alamouti Block STC CPM Modulation

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***Abstract*— We assume burst Alamouti coded STC CPM (MSK) modulation in MIMO channel as a particular example of *Nonlinear Multichannel Modulation (NMM)*. The NMM has multidimensional constellation waveforms. The major goal is to find suboptimal receiver signal preprocessing that could substantially reduce receiver decoding metric computation complexity with minor degradation of the STC performance. We introduce linear waveform subspace projector reducing the dimensionality and thus the complexity of the receiver signal processing. We analyze the impact of the subspace projector on the code performance in terms of its rank, determinant (diversity and coding gain) and bit error rate. We find an optimal subspace basis which does not reduce the rank of the code and which affects the determinant and bit error rate by the minimum possible amount.**

I. Introduction

The research activity in the area of Space-Time Coded (STC) modulations for Multiple-Input Multiple-Output (MIMO) communication systems attracted a lot of attention in recent years. A huge number of results is already available (see e.g. [1] and the references therein). However most of the results concentrated on *linear* modulation schemes (e.g. PSK, QAM). The *Nonlinear Multichannel Modulation (NMM),* due its inherent complexity, remains explored only sparsely. Several results on coding schemes and capacity can be found in [2], [3], [4].

A. Motivation

The NMM can be an very attractive option due the additional degree of freedom for forming the waveform shape and other properties (e.g. constant envelope and continuous phase). Particularly the c*onstant envelope* property of CPM modulation is extremely useful from the technological point of view on the transmitter side (e.g. the C-class amplifier). However, on the receiver side, we are typically constrained by the processing complexity (available DSP resources, etc.). The signal space representation of the NMM signal is multidimensional (per antenna) and can further increase an implementation complexity by increasing the dimensionality of the signal processing (e.g. the metric computation in Viterbi algorithm). An obvious goal is to reduce the signal dimensionality while preserving the properties of the original modulation as far as possible. This can reduce substantially the processing complexity at the receiver. There are generally two possible approaches — the linear and nonlinear projecting receiver preprocessor. This paper follows the linear dimensionality reducing preprocessing line. See [5] for the nonlinear preprocessing approach.

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B. Goals

We assume STC CPM (MSK) modulation in MIMO channel. The code we consider is a simple burst based Alamouti code applied on CPM [6]. The major goal is to find suboptimal receiver signal preprocessing that could substantially reduce receiver decoding metric computation complexity with minor degradation of the STC performance. We develop receiver linear preprocessing with constellation waveform subspace projection. This reduces the dimensionality and therefore signal processing complexity of the receiver (particularly the decoding metric computation). The projection basis is required to be *orthogonal* and *Nyquist* in order to facilitate the receiver metric computation (the folding condition, see Sec. IV-C).

We analyze the code performance in terms of its rank and determinant property (diversity and coding gain) and also actual bit error rate under realistic signal to noise ratio as a function of the particular subspace basis and we identify the optimal basis. We assume flat fading Rayleigh MIMO channel and perfect channel state information at the receiver.

II. System model and Definitions

This section briefly introduces the system model. The main point is the *multidimensional* (per antenna) constellation waveform space description of *nonlinear* modulation and corresponding waveform space channel model.

A. Nonlinear multichannel modulator

The NMM generates *nonlinearly* modulated signal at each transmitter antenna. The complex envelope signal on i-th transmit antenna is

$$s_i(t) = \sum_n g(q_{n,i}, t - nT_S) \quad (1)$$

where T_S is the symbol period, $g(q_{n,i}, t)$ is generally *nonlinear* modulation waveform, $q_{n,i} \in \{q^{(m)}\}_{m=1}^{M_q}$ is a channel symbol and n is its sequence number. The channel symbol depends on the modulator input data $c_{n,i} \in \{c^{(m)}\}_{m=1}^{M_c}$ and modulator state $\sigma_{n,i} \in \{\sigma^{(m)}\}_{m=1}^{M_\sigma}$ through time-invariant generally *non-linear* function $q_{n,i} = q_i(c_{n,i}, \sigma_{n,i})$. M_q, M_c, M_σ are alphabet (per-transmitter) sizes for channel symbol, input codeword, and modulator state respectively.

The function $q_i(c_{n,i}, \sigma_{n,i})$ forms a discrete part of the modulator. It can be fully described by the Finite State Machine (FSM) model. The function $g(q,t)$ forms an expansion part (discrete input, continuous waveform output) of the modulator. The set of all possible modulator expansion part output waveforms is $g(\bullet, t) \in \{g^{(m)}(t)\}_{m=1}^{M_q}$. The modulation functions are assumed to be Nyquist ones. This guarantees that the expansion part is memoryless. Modulators (discrete and expansion parts) in individual transmitter branches are independent and identical.

A proper choice of the discrete and expansion part of the modulator can be used to constrain waveform and memory behavior of the signal. The most typical waveform constraint is the constant envelope one. As an additional tool, the memory constraint can be imposed. The most typical one is the continuous signal constraint. As an practically important example, the CPM class of modulation uses both of the above mentioned.

The decomposition approach using FSM is essentially based on [7] with phase modulator replaced by a linear multi-D expansion. This however directly applies only to the full-space signal (no sub-space projection). See Sec. IV-C for the discussion of the expansion basis properties (particularly the Nyquist property) in the case of the sub-space projection.

B. Channel

The channel is considered to be frequency flat block-constant fading (N_T, N_R) MIMO channel with AWGN (Additive White Gaussian Noise). A received signal at k-th receive antenna is

$$x_k(t) = \sum_{i=1}^{N_T} h_{ki} s_i(t) + w_k(t) \qquad (2)$$

where h_{ki} are channel coefficients and $\{w_k(t)\}_{k=1}^{N_R}$ are IID zero mean rotationally invariant complex white Gaussian noise processes with power spectrum density $S_w(f) = 2N_0$. Channel coefficients are zero mean IID complex Gaussian random variables with unity variance $\mathrm{E}[|h_{ki}|^2] = 1$. We also form a $(N_R \times N_T)$ channel matrix $[\mathbf{H}]_{ki} = h_{ki}$.

C. Multidimensional constellation waveform space model

Arbitrary nonlinear modulation can be equivalently expressed using multidimensional waveform Euclidean signal space representation of the waveform corresponding to the n-th symbol at one antenna

$$g(q_{n,i}, t) = \mathbf{s}_{n,i}^T \mathbf{g}(t). \qquad (3)$$

The vector $\mathbf{s}_{n,i} = [s_{n,i,1}, \ldots, s_{n,i,N_s}]^T$ is the N_s dimensional signal space representation of the waveform (constellation

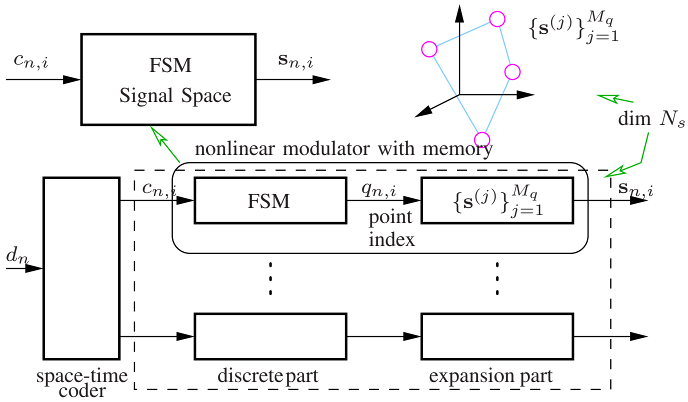

Fig. 1. Multidimensional constellation waveform space model of NMM.

point) and $\mathbf{g}(t) = [g_1(t), \ldots, g_{N_s}(t)]^T$ is vector of *orthonormal* and *Nyquist* basis functions. The space spanned by $\mathbf{g}(t)$ is called *constellation (waveform) space*. In a special case of linear modulation, the dimension is $N_s = 1$. For nonlinear modulation, this is however $N_s > 1$. The constellation points can be get e.g. by Gram-Schmidt procedure or by informationally equivalent decomposition [4]. The constellation points are selected by the output of the FSM $q_{n,i} = q_i(c_{n,i}, \sigma_{n,i})$ (Fig. 1).

An alternative to the orthonormal and Nyquist waveform space basis is the Laurent decomposition [8]. It also produces linear expansion of the waveform. However, it possesses one very important deficiency — the Laurent decomposition (including the tilted phase variants) *does not* produce orthonormal nor Nyquist basis. The orthonormality and Nyquist property is a very important feature from the point of view of the code design and also for the receiver signal processing (particularly the receiver metric evaluation).

The fact that the nonlinear modulation has waveform dimensionality $N_s > 1$ has a number of positive consequences. It opens additional dimensions that can be used for the code design increasing both the capacity and the potential diversity gain. See [9] and [4] for details.

D. Waveform Space channel model for NMM

The received signal at k-th receive antenna (dimensionality N_s per antenna) in signal space notation is

$$\mathbf{x}_{n,k} = \sum_{i=1}^{N_T} h_{ki} \mathbf{s}_{n,i} + \mathbf{w}_{n,k}. \qquad (4)$$

This can be written in a compact form using space-stacked vectors and matrices

$$\tilde{\mathbf{x}}_n = \tilde{\mathbf{H}} \tilde{\mathbf{s}}_n + \tilde{\mathbf{w}}_n \qquad (5)$$

where $\tilde{\mathbf{x}}_n = [\mathbf{x}_{n,1}^T, \ldots, \mathbf{x}_{n,N_R}^T]^T$ and similarly for $\tilde{\mathbf{s}}_n$ and $\tilde{\mathbf{w}}_n$. The stacked channel matrix has a *Kronecker* structure $\tilde{\mathbf{H}} = \mathbf{H} \otimes \mathbf{I}_{N_s}$ ($\otimes$ is a Kronecker product), see Fig. 2. Channel eigenmodes $\tilde{\mathbf{x}}_n = \tilde{\mathbf{V}}_1 \tilde{\mathbf{D}} \tilde{\mathbf{V}}_2^H \tilde{\mathbf{s}}_n + \tilde{\mathbf{w}}_n$ can be also shown to have a Kronecker structure $\tilde{\mathbf{V}}_1 = \mathbf{V}_1 \otimes \mathbf{I}_{N_s}$, $\tilde{\mathbf{V}}_2 = \mathbf{V}_2 \otimes \mathbf{I}_{N_s}$ $\tilde{\mathbf{D}} = \mathbf{D} \otimes \mathbf{I}_{N_s}$ where $\mathbf{H} = \mathbf{V}_1 \mathbf{D} \mathbf{V}_2^H$.

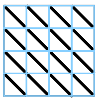

Fig. 2. Stacked channel matrix with Kronecker structure.

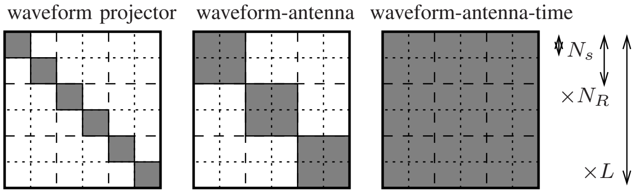

Fig. 3. Examples of the subspace projection matrix. $L = 3$, $N_R = 2$, $N_s = 2$.

III. Linear constellation waveform subspace projection

A linear constellation waveform subspace projector significantly simplifies the implementation of the receiver by lowering the dimensionality of the problem. However it affects the properties, namely the coding gain and diversity gain, of the code used. The detailed analysis can be found in [10]. Here we summarize the results.

A. Linear subspace projector

The linear subspace projector is a preprocessor operating on the received signal $\tilde{\mathbf{x}}_n$. In a first approach, it can operate in the *waveform* and *antenna space* dimensions. The projector output is $\tilde{\mathbf{y}}_n = \tilde{\mathbf{P}}_1 \tilde{\mathbf{x}}_n$ where $\tilde{\mathbf{P}}_1$ is the space-only projection matrix. The generalized variant is the projector operating jointly in *waveform-antenna-time space* dimensions. It operates on the received vectors stacked in all waveform space, antenna space and time domain $\tilde{\mathbf{y}} = \tilde{\mathbf{P}}_L \tilde{\mathbf{x}}$ where (and similarly for $\tilde{\mathbf{y}}$) $\tilde{\mathbf{x}} = [\tilde{\mathbf{x}}_1^T, \ldots, \tilde{\mathbf{x}}_L^T]^T$ is $\tilde{N} \times 1$ vector, $\tilde{N} = N_s N_R L$. The value L denotes the length of time observation window. The projection matrix is [11]

$$\tilde{\mathbf{P}}_L = \tilde{\mathbf{B}}(\tilde{\mathbf{B}}^H \tilde{\mathbf{B}})^{-1} \tilde{\mathbf{B}}^H \tag{6}$$

where $\tilde{\mathbf{B}} = [\tilde{\mathbf{b}}_1, \ldots, \tilde{\mathbf{b}}_{\tilde{r}}]$ is the $\tilde{N} \times \tilde{r}$ matrix of column-wise basis vectors of the $\tilde{r}$-dimensional constellation waveform subspace ($\tilde{r} < \tilde{N}$) the signal is projected to. Examples of the projection matrix are shown on Fig. 3.

In the waveform space and waveform-antenna space projector variants, the projection matrix can be constant for all orthogonal subspaces. We call this case a Kronecker projection matrix since the projection matrix can be expressed using the Kronecker product. In the waveform space variant (and similarly for waveform-antenna space), it is

$$\tilde{\mathbf{P}} = \mathbf{I}_{L N_R} \otimes \mathbf{P} \tag{7}$$

where $\mathbf{P}$ is the common waveform space projection $N_s \times N_s$ matrix with rank $r < N_s$.

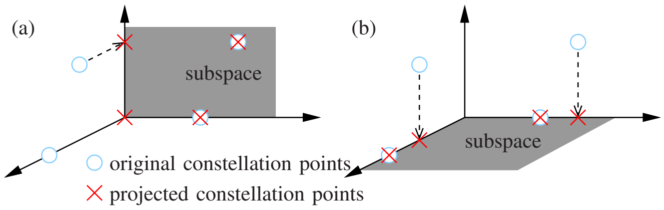

Fig. 4. Example of the Kronecker waveform space constellation projections (3 real dimensions into 2 real dimensions) with different minimum distance of the equivalent projected constellation points $\rho_{\min}(\{\tilde{\mathbf{u}}_n^{(m)}\}_m)$, $\rho_{\min}(\{\tilde{\mathbf{u}}_{n,(a)}^{(m)}\}_m) > \rho_{\min}(\{\tilde{\mathbf{u}}_{n,(b)}^{(m)}\}_m)$.

B. Code performance criteria affected by the projection

We assume given fixed transmitted NMM with known multidimensional constellation. We also assume given and fixed subspace dimension dictated by the receiver implementation complexity limits. For a given chosen subspace dimension $\tilde{r}$, we seek the matrix $\tilde{\mathbf{P}}$ maximizing the code performance criteria. For a Rayleigh MIMO flat channel, the performance criterion is typically based on *rank* and *determinant* of the inner product (Gram) matrix for codeword pairs differences [1].

In the simplest case of *Kronecker waveform only* projector, we can show that

$$\begin{aligned} \tilde{\mathbf{y}}_n &= (\mathbf{I}_{N_R} \otimes \mathbf{P})(\mathbf{H} \otimes \mathbf{I}_{N_s})\tilde{\mathbf{s}}_n + (\mathbf{I}_{N_R} \otimes \mathbf{P})\tilde{\mathbf{w}}_n \\ &= (\mathbf{H} \otimes \mathbf{I}_{N_s})(\mathbf{I}_{N_R} \otimes \mathbf{P})\tilde{\mathbf{s}}_n + \tilde{\mathbf{w}}_n' \\ &= \tilde{\mathbf{H}}\tilde{\mathbf{u}}_n + \tilde{\mathbf{w}}_n' \end{aligned} \tag{8}$$

where the new *equivalent projected transmitted constellation points* are $\tilde{\mathbf{u}}_n = [\mathbf{u}_{n,1}^T, \ldots, \mathbf{u}_{n,N_T}^T]^T$, $\mathbf{u}_{n,i} = \mathbf{P}\mathbf{s}_{n,i}$. Noise projection $\tilde{\mathbf{w}}_n' = (\mathbf{I}_{N_R} \otimes \mathbf{P})\tilde{\mathbf{w}}_n$ is IID Gaussian again.

Gram matrix for codeword pairs differences for original and projected constellations are $\mathbf{R}_{\mathbf{s}} = \Delta\mathbf{S}^H \Delta\mathbf{S}$, $\mathbf{R}_{\mathbf{u}} = \Delta\mathbf{U}^H \Delta\mathbf{U}$ where codeword (with length L_c) differences are $\Delta\mathbf{S} = \mathbf{S}^{(a)} - \mathbf{S}^{(b)}$,

$$\mathbf{S}^{(a)} = \begin{bmatrix} \mathbf{s}_{1,1} & \cdots & \mathbf{s}_{1,N_T} \\ \vdots & \ddots & \vdots \\ \mathbf{s}_{L_c,1} & \cdots & \mathbf{s}_{L_c,N_T} \end{bmatrix}. \tag{9}$$

$\Delta\mathbf{U}$ is defined similarly. Clearly, it is $\Delta\mathbf{U} = (\mathbf{I}_{L_c} \otimes \mathbf{P})\Delta\mathbf{S}$ and as a consequence

$$\mathbf{R}_{\mathbf{u}} = \Delta\mathbf{S}^H (\mathbf{I}_{L_c} \otimes \mathbf{P})\Delta\mathbf{S}. \tag{10}$$

The Kronecker waveform space only projector can be simply optimized by considering a new equivalent projected constellation set of transmitted codewords $\{\tilde{\mathbf{u}}_n^{(m)}\}_m$. Fig. 4 shows two examples of the constellation projection in the Kronecker waveform only case.

IV. Subspace Projector Design for Burst Alamouti Coded CPM

A. Burst Alamouti coded CPM

The CPM modulation posses inherently the memory which plays an important role in guaranteeing the continuity of its phase. This fact makes all attempts of constructing the Space-Time Trellis Code (ST-TC) a difficult task, see [3], [2]. The major problem stands in the necessity to construct two concatenated trellises with given code performance criteria (coding and diversity gain). The situation for Space-Time Block Code (ST-BC) is somewhat easier. However the block nature of the code could easily damage the continuity of the phase at the block end. A relatively easy solution to this problem is possible for the *burst* based transmission. The short signal burst can be coded on the full burst level using one of the codes based on the Orthogonal Design (OD), [6]. The simplest example of OD is Alamouti code [12].

As an example application of the subspace projection, we will adopt the approach of [6] and apply the Alamouti code on the signal entities being the bursts of the CPM modulated signal. The scalar CPM modulated signal $s(t)$ is split into two parts (sub-bursts) $s_{B1}(t), s_{B2}(t)$. See the [6] for the discussion of the tail effects, burst length impact and the detection/decoding algorithm. The modulated signal for two transmit antennas is ($N_T = 2$)

$$\mathbf{s}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} s_{B1}(t) & -s^*_{B2}(t) \\ s_{B2}(t) & s^*_{B1}(t) \end{bmatrix}. \quad (11)$$

For our purposes, we assume the burst length be long enough to allow correct evaluation of the CPM trellis on the code performance.

B. Code performance criteria

We will investigate how sub-space projector affects the code coding gain and diversity gain performance measured in terms of determinant and rank criteria [1]. The full-space unmodified signal has $\det(\mathbf{R_s})$, $\mathrm{rank}(\mathbf{R_s})$ and the equivalent projected sub-space code has $\det(\mathbf{R_u})$, $\mathrm{rank}(\mathbf{R_u})$. The obvious goal is to find such sub-space that the determinant and rank of Gram matrix for codeword pairs differences becomes affected as low as possible.

C. Waveform space of the CPM modulation

As an example application of the projector, we consider a MSK modulation as a special, practically important case of CPM. The MSK modulation has $N_s = 2$ (complex) dimensional waveform space for modulation function corresponding to one channel symbol. The non-orthogonal basis is

$$\{\xi_1(t), \xi_2(t)\} = \left\{\exp(j2\pi\kappa t)/\sqrt{T_S}, \exp(-j2\pi\kappa t)/\sqrt{T_S}\right\}, \quad (12)$$

$t \in [0, T_S)$ where T_S is the symbol period and the modulation index is $\kappa = 1/2$. We use Gram-Schmidt process to get the orthonormal basis

$$\zeta_1(t) = \xi_1(t), \quad (13)$$

$$\zeta_2(t) = (\xi_2(t) + 2j\xi_1(t)/\pi)/\sqrt{1 - 4/\pi^2}. \quad (14)$$

It very important to stress that such a basis is *orthonormal* and *Nyquist* for individual modulation functions in the sequence. It plays an important role in receiver algorithms, the Nyquist condition guarantees that the decoder metric is a sum of individual contributions for individual channel symbols (folding condition). Without that, the receiver processing becomes very awkward. The well-know Laurent CPM decomposition (including the tilted phase variant) [8] does *not* produce *Nyquist* basis.

There is $M_s = 8$ distinct waveforms of the MSK with waveform space representation ($a_1 = 2/\pi$, $a_2 = \sqrt{1 - 4/\pi^2}$)

$$\left\{\mathbf{s}^{(1)}, \ldots, \mathbf{s}^{(M_s)}\right\} = \left\{[1,0]^T, [j,0]^T, [-1,0]^T, [-j,0]^T, [-ja_1, a_2]^T, [a_1, ja_2]^T, [ja_1, -a_2]^T, [-a_1, -ja_2]^T\right\}. \quad (15)$$

D. Waveform sub-space

The receiver waveform-only Kronecker projector has subspace dimension $N_r = 1$. The sub-space basis is defined as a parametric composition from full-space basis $\mathbf{b}_1 = [1, be^{j\beta}]^T$. Based on this, we can find the projector matrix $\tilde{\mathbf{P}}$, (7) and consequently the Gram matrices of full-space and sub-space equivalent constellation $\mathbf{R_s} = \Delta\mathbf{S}^H\Delta\mathbf{S}$, $\mathbf{R_u} = \Delta\mathbf{U}^H\Delta\mathbf{U}$.

E. Sub-space numerical optimization

We have evaluated numerically the determinant and the rank of the full-space and the sub-space equivalent constellation Gram matrices $\mathbf{R_s}$, $\mathbf{R_u}$ parametrized by b, β in order to find the optimum projection. The behavior of $\min(\det(\mathbf{R_u}))$ is captured at Fig. 5. We clearly see, that there is an optimal value of b, β minimizing the impact on the determinant and therefore on the coding gain $b_{\mathrm{O1}} \approx 0.6, \beta_{\mathrm{O1}} = \pi$. The *rank* remains *unaffected* by the projection for any b, β, i.e. $\mathrm{rank}(\mathbf{R_u}) = \mathrm{rank}(\mathbf{R_s}) = 2$. It has rank 2 as we expect for the Alamouti code.

A computer simulation of the bit error rate performance was performed in order to find its dependence on the projector parameters (Fig 7). The optimal values found by this procedure are $b_{\mathrm{O2}} = 1, \beta_{\mathrm{O2}} = \pi$.

V. Discussion of the Results and Conclusions

The analysis and numerical results shows quite optimistic results. We have reduced the dimensionality from $N_s = 2$ to $N_r = 1$ which substantially reduces the receiver signal processing complexity. The rank of the codeword matrix (the diversity gain) remains unaffected. This is an expected result, since the Alamouti code is applied on all dimensions of the constellation.

The determinant of the codeword matrix describes essentially the free distance properties of the code. The coding gain of the burst Alamouti STC CPM is nonzero, unlike for the linear memoryless schemes (like PSK), due to a presence of the inherent CPM memory. The minimum of the determinant affects the asymptotic coding gain behaviour. The coding gain is reduced by the projector only by a ratio $\approx 3.5/4$ in the case of the subspace basis optimal w.r.t. the determinant criterion, i.e. $b_{\mathrm{O1}} \approx 0.6, \beta_{\mathrm{O1}} = \pi$. The bit error rate performance get from the simulations under practically relevant signal to noise ratio shows slightly different optimal subspace basis

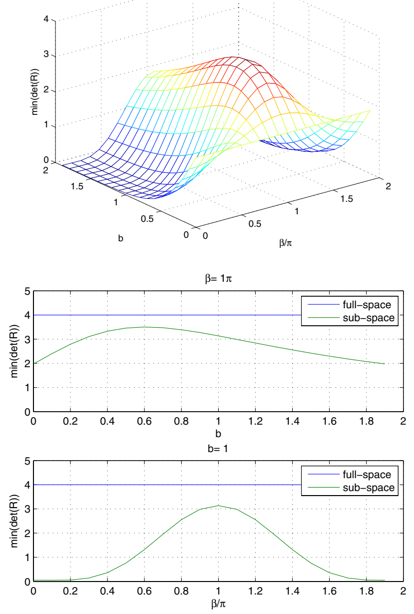

Fig. 5. The dependence of $\min(\det(\mathbf{R_u}))$ on b, β.

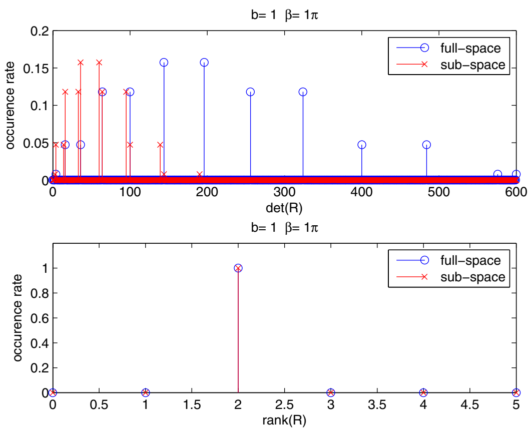

Fig. 6. The occurrence rate (normalized histogram) of the rank and the determinant for all data sequences in the case of optimal sub-space basis.

$b_{O2} = 1, \beta_{O2} = \pi$. The reason for that seems to be a change of the distance profile of the modulated signal caused by the projection. The occurrence rates (normalized histograms) of the rank and the determinant for the optimal (w.r.t. the bit error rate) sub-space basis ($b_{O2} = 1, \beta_{O2} = \pi$) for all data sequences are shown on Fig. 6. It shows the change of the $\det(\mathbf{R})$ profile (the occurrence rate over all data sequences) caused by the projection. Although the $\min(\det(\mathbf{R}))$ is affected only negligibly, the main mass of the profile is shifted towards the smaller values for the projected signal. This explains the slightly different optimization results and also slightly higher degradation of the bit error rate than the one that would had been expected from the $\min(\det(\mathbf{R_u}))$ (see Fig. 7).

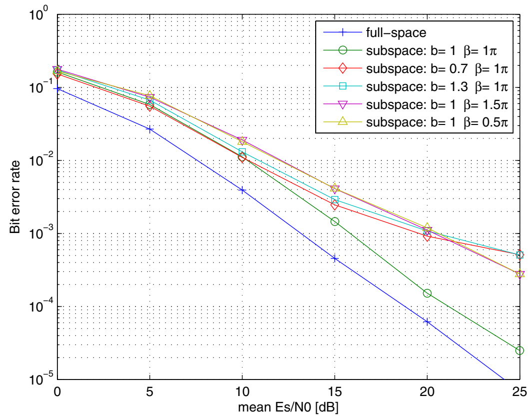

Fig. 7. Bit error rate for various sub-space settings in Rayleigh flat fading (2×1) channel.

A proper choice of the subspace basis is found to be very important. Other than the optimal one can reduce the coding gain very substantially. There is also clearly visible the error floor behaviour for non-optimal value of b. This is likely caused by the self-noise stemming from the Alamouti orthogonality violation caused by the projection. However this phenomenon needs further investigation. The application of non-optimal angle β changes the slope of the bit error rate curve but does not seem to produce the error floor (see Fig. 7).

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