Wireless-aware Network Coding: Solving a Puzzle in Acyclic Multi-stage Cloud Networks

Tomas Uricar, Tomas Hynek, Pavel Prochazka and Jan Sykora
E-mail:{uricatom, hynektom, prochp10, Jan.Sykora}@fel.cvut.cz
Czech Technical University in Prague, Czech Republic

Abstract / Summary

An immense number of mutually interacting nodes increases the complexity of wireless cloud networks design/analysis. This complexity is driven by an enormous amount of degrees of freedom in the system, which are available even if a logical network structure is clearly defined and channel characteristics are known (at least locally). These degrees of freedom comprise distribution of information through the cloud, terminal(s) & relay(s) node processing (modulation, coding, relaying techniques) etc. In this paper we try to develop a framework for analysis/design of such cloud networks. We propose a partitioning of the generally complex cloud analysis/design problem into three separate tasks, which can be analysed (to some extent) independently. By identifying the mutual relations among these three tasks (and the outer conditions) the entire cloud network can be designed/analysed, similarly as a large picture can be created by assembling interlocking jigsaw puzzle pieces. The idea of Wireless-aware Network Coding will be identified as a cornerstone of our framework, providing a primary insight into the information flows passing through the cloud network. This principle will be demonstrated in an analysis of a simple single-stage system.

1 Introduction

The basic idea of Wireless (Physical-Layer) Network Coding (WNC) was built upon the principles of conventional Network Coding (NC) [1], incorporating the inherent properties of wireless channels (broadcast nature, instant combining of electromagnetic waves, etc.). While the NC principles can be relatively easily adopted in general multi-node multi-source networks (see e.g. [2]), the research in the field of WNC still lags behind, having the majority of results limited to basic (and well defined) network structures. This is mainly due to the immense number of degrees of freedom, typically available in general wireless multi-node multi-source networks.

A large number of mutually interacting entities undoubtedly increases the complexity of analysis/design of multi-node network. Obviously, even when the network structure is clearly defined and all receiving nodes know perfectly characteristics of the observed (wireless) channels, numerous degrees of freedom still remain available for the network design. These degrees of freedom include distribution of information flows throughout the cloud, choice of relaying strategies at particular relay (cloud) nodes (e.g. Joint Decode & Forward (JDF), Hierarchical Decode & Forward (HDF), Amplify & Forward (AF), etc. [3–6]), modulation and coding schemes at source nodes and decoding of desired data from the received information streams at destinations, just to name a few.

It is obvious that many of the aforementioned degrees of freedom mutually influence each other, resulting in a highly complex design problem. Numerous outer conditions, either inherently laid by the nature (e.g. random network connectivity, channel states etc.) or by the manufacturers (e.g. node/system capabilities, etc.) must be also taken into account to meet a required utility metric (throughput/outage/latency etc.) in a given environment.

Figure 1: Puzzle framework.

The goal of this paper is to introduce a unified design/analysis framework for a general wireless acyclic multi-stage cloud network. The principal idea of the proposed framework is based on a partitioning of a general complex problem into several smaller elements (named tasks) and identification of mutual interfaces among these elements (and the outer conditions). By identifying these mutual relations, the entire cloud network can be designed/analysed, similarly as a large picture can be created by assembling interlocking jigsaw puzzle pieces. Even though the particular pieces could be designed separately, they must interlock perfectly together to establish reliable communication over the network. The proposed modular framework should help to answer questions like: How should the cloud nodes process the information received from sources to guarantee a reliable decoding of the desired data at all destinations? How could the choice of the cloud node relaying strategy affect the overall throughput? How would some change within the system (connectivity
pattern, channel gains, internal cloud node(s) failure, particular processing, etc.) influence the feasibility (or performance) of some particular cloud transmission strategy? We are perfectly aware that a fully comprehensive framework, able to answer similar questions is out of the scope of this paper. Nonetheless, we would like to identify at least some fundamental relations within the framework, giving some initial hints for future work. The main goals of the paper can be summarized as follows:

- Definition of the basic framework elements (Fig.1).
- Introduction of a tool for an analysis of information flows in a general acyclic multi-stage cloud network.
- Example demonstration of the framework principles: Evaluation of achievable throughput in a single-stage cloud network.

2 Multi-stage Acyclic Network

A general acyclic multi-stage wireless network is depicted in Fig.2. This network embraces multiple sources and destinations and a large number of relay nodes that form a wireless cloud. The relays in the cloud are assumed to be clustered into several separate layers (stages). Nodes within each such layer are able to receive only transmissions from the nodes in the neighbouring layer (1st layer receives information directly from sources) and hence the cloud communication is performed sequentially on a per-layer (per-stage) basis. This assumption guarantees that the whole network is cycle free.

![General multi-stage acyclic network.](image)

**Figure 2:** General multi-stage acyclic network.

2.1 Principles of Puzzle Framework

Adoption of the relaying techniques and WNC principles in wireless networks provides the tools for performance improvement beyond the limits induced by conventional routing approach. Unfortunately, the potential performance leap is tightly bound with an increased complexity of the analysis and design of such wireless cloud network, which grows exponentially with the number of network nodes. The state-of-the-art research focuses on very primitive (typically 3–5 nodes) network structures, whereas only some ad-hoc solutions are available for general multi-node wireless cloud networks. This fact underscores the necessity of a unified design/analysis framework.

There are three to some extend standalone research problems, which must be covered by the framework (Fig.1):

1. Distribution of information flows in the network.
2. Terminal nodes processing.
   - (a) Source node modulation, coding and signal processing.
   - (b) Destination node decoding algorithms.
3. Cloud nodes processing and relaying strategies.

In the following sections we will briefly discuss the above defined framework elements. The emphasis will be put mainly on the first element, i.e. the distribution of information flows in the network. Only a thorough understanding of this problem can help to identify the optimal methods for a transmission of source information through cloud nodes towards the intended destinations. A particular terminal and cloud node processing can be always built on top of a specific information flow strategy and it should serve mainly for an optimization of the strategy for a given utility metric (e.g. sum-rate, outage etc.). Some particular properties of the framework will be demonstrated on a primitive cloud network model – 2-way relay channel (2-WRC) – see Fig.3.

![2-way relay channel.](image)

**Figure 3:** 2-way relay channel. Two nodes $S_A/D_B$, $S_B/D_A$ share a single cloud node (relay $R$).

2.1.1 Wireless-aware Network Coding

The first and fundamental problem which must be covered by the framework is the way how the cloud nodes distribute the information from sources to all intended destinations. Contrary to the conventional routing approach, where the intermediate cloud nodes only decode and forward separate source information streams, in the WNC-based network the cloud nodes can process directly the compound (hierarchical) information (see e.g. [3, 6]), without a separate decoding of individual source data. On the other hand, the final destinations in the WNC network require a specific form of side-information (Hierarchical Side Information - HSI [4]) to be able to decode the desired information from these hierarchical data streams.

To harness all the merits of wireless communication channels, the underlying NC principle must respect (and embrace) all its specific characteristics (broadcast nature, inherent combining of electromagnetic waves, partial information reception etc.) and hence the term **Wireless-aware Network Coding** (WaNC) fully describes the nature of this framework element. The WaNC principle focuses only on the flow of information streams through the network nodes, regardless of particular cloud and terminal node processing. It serves mainly for an identification of admissible information flow strategies, i.e. the strategies which will guarantee a decodability of desired information at all destination nodes.
If a connectivity pattern between the nodes in the acyclic multi-stage network is defined, WaNC can identify the set of all information flow strategies which are eligible to deliver the data to all required destinations. In the multi-stage network this can be done step-by-step for each layer of relays, as illustrated in Fig.4. For the given input data streams (defined by the connectivity to the relays in the preceding layer) each relay is able to identify a set of possible output data streams to the next layer. Note that by \( d_{ij} \) we denote a hierarchical function \( f(d_i, d_j) \) of data from sources \( S_i \) and \( S_j \). In this way the information streams from source nodes can be pursued through the cloud layers towards the destinations, where the set of feasible information flow strategies can be identified. Although this can be done by hand for some basic cloud network structures, investigation of some forward-backward search algorithm for identification of the set of feasible information flow strategies (on a graph description of the cloud network) would be desirable.

Note that in the 2-WRC (Fig.3) the set of feasible strategies includes only the conventional routing (relay sends separately both received source data streams \( d_A, d_B \)) and WNC-based strategies (only the compound hierarchical information \( d_{AB} \) is broadcast by the relay and both destinations can decode thanks to the availability of HSI – see e.g. [4]). The strategies where only one source data (either \( d_A \) or \( d_B \)) are sent by the relay cannot guarantee that both destinations receive the desired data and hence these strategies would be banned in the 2-WRC.

The set of feasible information flow strategies can serve as a basis for analysis of fundamental performance limits and also it can be used to identify the reference scenarios (e.g. conventional routing is always a feasible strategy). The WaNC answers only the question what are the possibilities for providing end-to-end source-to-destination connection. It does not define the optimal strategy in any sense, since it does not cover either the interactions with the outer conditions (channel characteristics etc.), or the particular node processing strategies (modulation/coding).

### 2.1.2 Terminal Node Processing

Particular performance (and feasibility) of any of the information flow strategy identified by WaNC would strongly depend on a particular source and destination node processing. This framework element should answer the questions like: How should the sources transmit the information streams to be reliably and effectively intercepted by the cloud nodes? How can the required information be retrieved from the received signals (separate data streams, hierarchical data, HSI) at destination nodes?

This framework element defines the source transmission rates, modulation/coding algorithms, together with decoding algorithms for destination nodes (including the decoding metrics and joint processing of hierarchical and HSI data streams). It should also reveal an impact of the processing at the sources and destination on the overall performance/utility metric and thus provide a valuable insight to the design of suitable physical layer processing algorithms. Later we will show that mainly a proper determination of source transmission rates (for a given outer conditions) is crucial for optimal sum-rate performance of any admissible WaNC flow strategy.

### 2.1.3 Cloud Node Processing

This framework element covers the particular physical layer processing algorithms performed by the cloud nodes. Apart of the modulation/coding algorithms it includes implementation of relaying strategies (AF/JDF/HDF) at cloud nodes, strategies for processing of partial HSI, algorithms for processing of hierarchical information streams (decoding, re-encoding and forwarding) and broadcast strategies for transmission of information towards the subsequent cloud layers (or destinations). Contrary to the source and destination nodes, the relays are assumed to be capable to cooperate (although possibly in a very limited way). This opens a space for advance and sophisticated signal processing techniques, including the distributed decision makings, decentralized synchronisation, advanced broadcast strategies etc.

Since there is only a single one cloud node in the 2-WRC, the implementation of WaNC strategies reduces there to a choice of conventional relaying strategy (AF, JDF, HDF). The analysis of the problem will become obviously much more complicated when more relay nodes are available in the network (even if these relays are clustered in a single layer). Numerous feasible WaNC strategies can be identified even in a simple 2-source 2-relay 2-destination cloud network, where the overall sum-rate performance would greatly depend on a specific combination of relay node strategies.

### 2.1.4 Mutual Relations among Framework Elements and Outer Conditions

It is apparent that the outer conditions (e.g. random network connectivity, channel gains, or node/system capabilities) significantly interacts with all the defined framework elements. These outer conditions defines the fundamental constraints and limits for the network design and hence...
they strongly influence the achievable performance/utility metric. We believe that only a thorough analysis of the mutual interfaces between all the pieces of the framework puzzle (and the outer conditions) can lead to a deep understanding of the underlying mutual interactions and relations in the wireless cloud network.

The enormous amount of degrees of freedom available in the wireless cloud network is apparent even in the basic 2-WRC scenario with a single cloud node. Although there are only a few WaNC strategies available, a vast number of scientific and research papers which focuses on this basic scenario substantiates the complexity of the analysis/design of general wireless cloud networks. This justifies the need for a unified design/analysis framework. In the following section we will demonstrate how the feasible WaNC strategies can be identified in a simple single-layer cloud network and show how the particular choice of source rates and relaying strategies can influence the achievable sum-rate performance.

3 Basic Example

We will illustrate some basic features of the proposed puzzle framework in a simple 2-relay, single-layer cloud network example. Assume a 2-source, 2-relay, 2-destination network as in Fig.5. Two independent sources $S_A, S_B$ want to deliver their information to their intended destinations $D_A, D_B$ (respectively). There are no direct source-destination links and hence a help of the relays $R_1, R_2$ is required. Relay nodes constitute single-stage acyclic cloud. For the sake of simplicity we assume a symmetrical network and hence also a symmetric source information rates $r_A = r_B = r$. Channels connecting each source with the nearest relay has SNR $\gamma_1$, links to the remote relays $\alpha \gamma_1$ ($\alpha \in \{0, 1\}$) and relay-destination links $\gamma_2$.

![Figure 5: Single-stage 2S-2R butterfly network.](image)

In this network, the set of all feasible WaNC strategies will include the following strategies (from the perspective of one destination):

1. Both separate data streams (or at least the single one intended data stream) are received.
2. Hierarchical data stream and corresponding HSI (the unintended data stream) are received.
3. Two different hierarchical streams (which allow a recovery of both data streams) are received.

There are numerous relaying strategies which can be used by the relay nodes to realize one of the above mentioned WaNC strategies. Since both destinations have connectivity to both relay nodes the feasible strategy can be implemented even when one of the relay nodes remains silent. The set of feasible relaying strategies for the whole network is a Cartesian product of each individual relay strategy set $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2$. Elements of this set are given by pairs of the particular relay strategies.

In the presented example we assume the following relaying strategies:

- **JDF** - relay decodes both source data and broadcasts the decoded messages in two time slots as an ordered pair to both destinations.
- **HDF$_i$** - relay decodes directly the hierarchical function and broadcasts it to both destinations in a single one time slot. Relays use HDF maps, denoted HDF$_1$, HDF$_2$ respectively, such that they form an invertible observation at each destination.
- **SDF** - relay decodes only the stronger source signal and broadcasts it to both destinations.
- **SDF weak** - similarly as the previous strategy, but each relay decodes and broadcasts only the weaker source.
- **"Do-nothing" (-)** - relay remains silent.

The Cartesian product (i.e. global strategy set $\mathcal{S}$) is depicted in Tab.1, where N/A symbol denotes the "nonsense" strategy (banned by WaNC).

For a given combination of relaying strategies, particular source/destination processing and outer conditions (channel gains), we are able to evaluate the performance of the network. In this example we will assume that the optimal modulation/coding strategy are exploited at all nodes and hence a rate equal to the channel capacity $C(\gamma) = \log_2(1 + \gamma)$ can be achieved on any link with SNR $\gamma$. To compare the performance of the strategies, we will use an approach similar to [3] to evaluate the achievable sum-rate performance. The sum-rate will be defined as a total information exchanged between source-destination pairs during one communication round (for more details see [3]).

The optimal source rates, maximizing the sum-rate are given by Eqs.(1a)-(1e):

\[
\begin{align*}
    r_1 & = \min \left\{ \frac{1}{2} C((\alpha + 1)\gamma_1), C(\alpha \gamma_1) \right\} \quad (1a) \\
    r_2 & = \frac{1}{2} \left\{ C(\gamma_1) + C \left( \frac{\gamma_1}{1 + \alpha \gamma_1} \right) \right\} \quad (1b) \\
    r_3 & = \frac{1}{2} \left\{ C(\alpha \gamma_1) + C \left( \frac{\alpha \gamma_1}{1 + \gamma_1} \right) \right\} \quad (1c) \\
    r_4 & = C(\alpha \gamma_1) \quad (1d) \\
    r_5 & = \min \{ r_2, r_4 \} \quad (1e)
\end{align*}
\]

Example MAC capacity regions for the relay observation are depicted in Fig.6. Detailed derivation of these rates is beyond the scope of this paper. The achievable sum-rates of all the strategies (the outer conditions are given by $\{\alpha_1, \gamma_2, \gamma_3\}$) are listed in Tab.1, where $r_i$ are the source rates defined in Eqs.(1a)-(1e). Sum-rates for $\gamma_1 = \gamma_2 = 10$dB as a function of $\alpha$ are depicted for all admissible strategy pairs in Fig.7. AF strategy is also depicted as a reference scenario.
is a partitioning of the exponentially complex problem. This paper presents the initial idea of the design/analysis framework for a general complex acyclic wireless cloud based networks. The fundamental idea of the framework is a partitioning of the exponentially complex problem (WNC-based cloud network analysis) into three smaller (and thus easily tractable) elements – Wireless-aware Network Coding, Terminal Node Processing and Cloud Node Processing (Fig.1). Identification of interfaces among all the framework elements should serve for better understanding of internal relations and mutual interactions within the complicated network. Thorough and detailed analysis of all the related mutual interfaces among the framework elements are beyond the scope of this paper and remain the subject for a future work.

5 Acknowledgement

Work in this paper was supported by the European Science Foundation through FP7-ICT DIWINIE project, the Grant Agency of the Czech Republic, grant 102/09/1624, the MEYS of the Czech Republic grant LD12062 and by the Grant Agency of the Czech Technical University in Prague, Grant no. SGS13/083/OHK3/1T/13.

References